Algebra 2
1.1 Domain, Range & End Behavior

Representing an Interval on a Number Line.

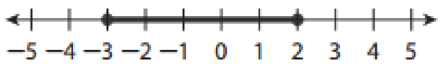
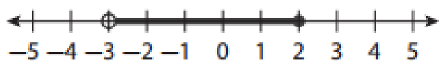
Description of interval	Type of interval	Inequality	Set notation	Interval notation
All real numbers from *a* to *b*, including *a* and *b*.	Finite	$a \le x \le b$	$\{x \mid a \le x \le b\}$	$[a, b]$
All real numbers greater than *a*	Infinite	$x > a$	$\{x \mid x > a\}$	$(a, +\infty)$
All real numbers less than or equal to *a*	Infinite	$x \le a$	$\{x \mid x \le a\}$	$(-\infty, a]$

Set Builder Notation:
| stands for "such that"
$\in$ stands for "is an element of"
$\mathbb{R}$ stands for "all real numbers"
ex) {x|x>3} which represents all x-values that are greater than 3.
ex) {y|y$\in\mathbb{R}$} which represents all y-values that are in the set of real numbers.

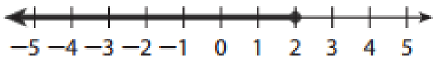
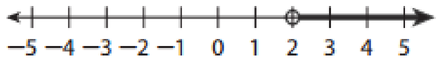
Interval Notation:
[ ] Use square brackets if including the endpoint.

( ) Use parenthesis if NOT including the endpoint.

Ⓐ Complete the table by writing the finite interval shown on each number line as an inequality, using set notation, and using interval notation.

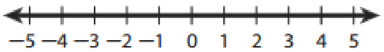
Finite Interval		
Inequality	$-3 \le x \le 2$	$-3 < x \le 2$
Set Notation	$\{x \mid -3 \le x \le 2\}$	$\{x \mid -3 < x \le 2\}$
Interval Notation	$[-3, 2]$	$(-3, 2]$

(B) Complete the table by writing the infinite interval shown on each number line as an inequality, using set notation, and using interval notation.

Infinite Interval		
Inequality	$x \le 2$	$x > 2$
Set Notation	$\{x \mid x \le 2\}$	$\{x \mid x > 2\}$
Interval Notation	$(-\infty, 2]$	$(2, \infty)$

Reflect

1. Consider the interval shown on the number line.



a. Represent the interval using interval notation. $(-\infty, \infty)$

b. What numbers are in this interval? $\mathbb{R}$

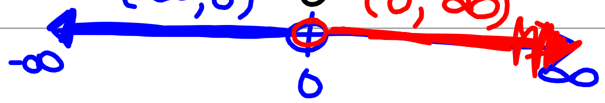
2. What do the intervals [0, 5], [0, 5), and (0, 5) have in common? What makes them different?

All are between 0 & 5 but some include the endpoints & some don't

3. Discussion The symbol ∪ represents the *union* of two sets. What do you think the notation $(-\infty, 0) \cup (0, +\infty)$ represents?

All #'s except 0

$(-\infty, 0)$ ∪ $(0, \infty)$



Domain: the set of x-values that can be put into a function.

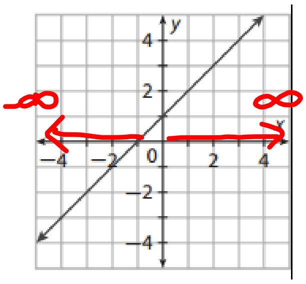
Range: the set of y-values [f(x)] that come out of a function.

End Behavior: describes what happens to the y-values as the x-values increase or decrease without bound.

Always written like: As $x \to \infty$, $f(x) \to$ ______ (y)

As $x \to -\infty$, $f(x) \to$ ______

Example: Write the end behavior of the following graph.

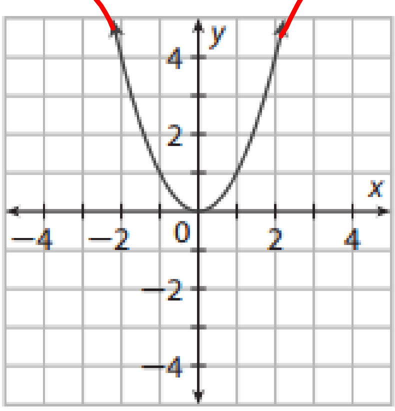


As $x \to \infty$, $f(x) \to \infty$

As $x \to -\infty$, $f(x) \to -\infty$

Example 1 Write the domain and the range of the function as an inequality, using set notation, and using interval notation. Also describe the end behavior of the function.

(A) The graph of the quadratic function $f(x) = x^2$ is shown.



Domain:

Inequality: $-\infty < x < \infty$

Set Notation: $\{x \mid -\infty < x < \infty\}$

Interval Notation: $(-\infty, \infty)$

Range:

Inequality: $y \geq 0$

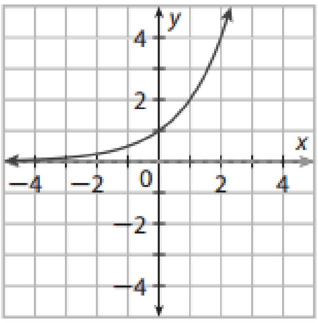
Set Notation: $\{y \mid y \geq 0\}$

Interval Notation: $[0, \infty)$

End Behavior: As $x \to \infty$, $f(x) \to \infty$

As $x \to -\infty$, $f(x) \to \infty$

(B) The graph of the exponential function $f(x) = 2^x$ is shown.



Domain: x's

Inequality: $-\infty < x < \infty$

Set Notation: $\{x \mid -\infty < x < \infty\}$

Interval Notation:

$(-\infty, \infty)$

Range: y's

Inequality: $0 < y < \infty$

Set Notation: $\{y \mid 0 < y < \infty\}$

Interval Notation:

$(0, \infty)$

End Behavior: As $x \to \infty$, $f(x) \to \infty$
As $x \to -\infty$, $f(x) \to 0$

Graphing a Linear Function on a Restricted Domain

$\mathbb{R}$

Unless otherwise stated, a function is assumed to have a domain consisting of all real numbers

$(-\infty, \infty)$

Sometimes a function may have a restricted domain. If the rule for a function and its restricted domain are given, you can draw its graph and then identify its range.

Example 2 **For the given function and domain, draw the graph and identify the range using the same notation as the given domain.**

(A) $f(x) = \frac{3}{4}x + 2$ with domain $[-4, 4]$

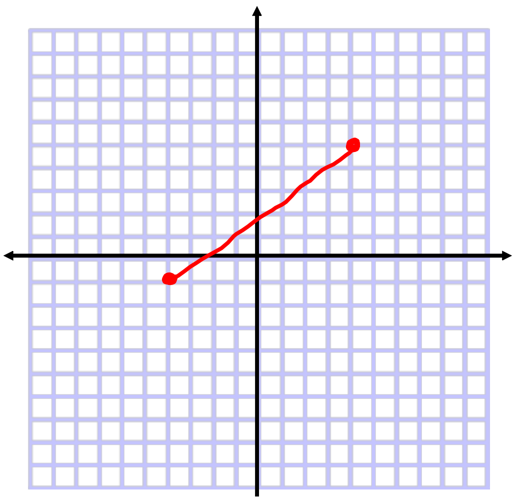
-4

$y = mx + b$

$x = -4 \le x \le 4$

$\frac{3}{4} \cdot -4 + 2$
$-3 + 2$
-1
$(-4, -1)$

$\frac{3}{4} \cdot 4 + 2$
$3 + 2$
5
$(4, 5)$



how does the graph change if the domain is $(-4, 4)$ instead of $[-4, 4]$?
open dots as endpts.

(B) $f(x) = -x - 2$ with domain $\{x|x > -3\}$

$-(-3)-2$

$3-2$

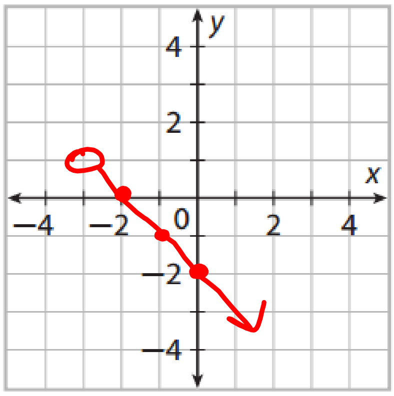
1

$(-3, 1)$

$m = \frac{-1}{1}$

$-0-2 = -2$

$(0, -2)$



In Part B, what is the end behavior as x increases without bound? Why can't you talk about the end behavior as x decreases without bound?

As $x \to \infty$, $f(x) \to -\infty$

b/c the xs are restricted

Your Turn

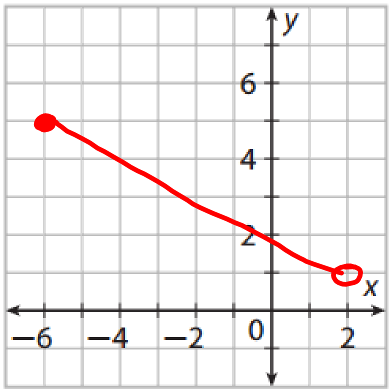
For the given function and domain, draw the graph and identify the range using the same notation as the given domain.

$f(x) = -\frac{1}{2}x + 2$ with domain $-6 \leq x < 2$

• $(-6, 5)$ $\quad -\frac{1}{2} \cdot -6 + 2 = 5$

○ $(2, 1)$ $\quad -\frac{1}{2} \cdot 2 + 2 = 1$

Range: $1 < y \leq 5$



Explain 3 Modeling with a Linear Function

Recall that when a real-world situation involves a constant rate of change, a linear function is a reasonable model for the situation. The situation may require restricting the function's domain.

Example 3 **Write a function that models the given situation. Determine a domain from the situation, graph the function using that domain, and identify the range.**

(A) Joyce jogs at a rate of 1 mile every 10 minutes for a total of 40 minutes. (Use inequalities for the domain and range of the function that models this situation.)

$D = R \cdot T$

$\frac{1}{10} = \frac{R \cdot 10}{10}$

$\frac{1}{10} = R = \text{slope}$

(t, d)

$(0, 0)$

$(40, 4)$

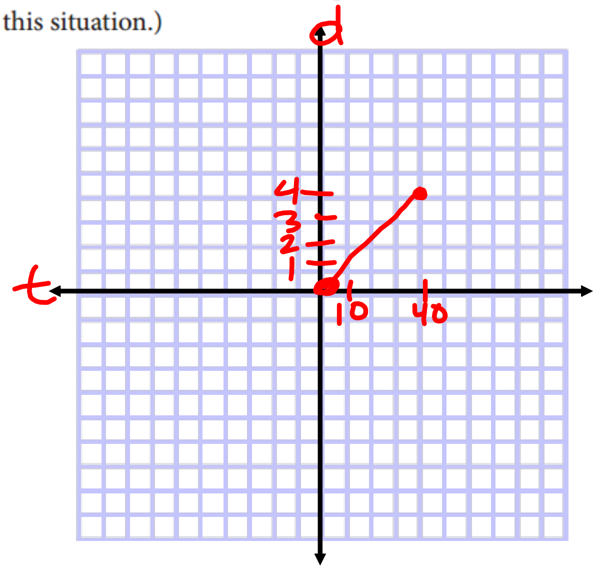
Domain $0 \le t \le 40$

$D = R \cdot T$

$4 = \frac{1}{10} \cdot 40$

Range

$0 \le d \le 4$



(B) A candle 6 inches high burns at a rate of 1 inch every 2 hours for 5 hours. (Use interval notation for the domain and range of the function that models this situation.)

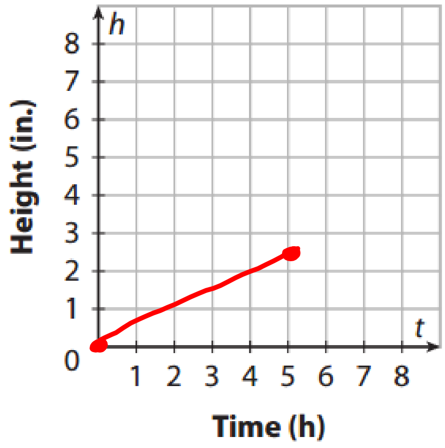
The candle's burning rate is $1/2$ in./h.

The candle's height h (in inches) at any time t (in hours) is modeled by $h(t) = \frac{1}{2}t$.

Since the candle burns for 5 hours, the domain is restricted to the interval $[0, \boxed{5}]$.

$\frac{1}{2} \cdot 5 = \frac{5}{2} = 2.5$

The range is $[0, 2.5]$



Homework

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