Math Analysis

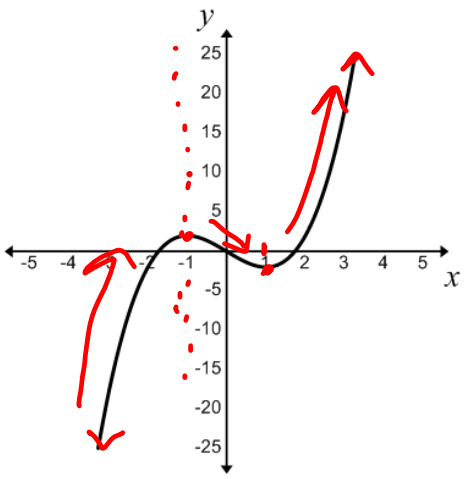
1.3 More on Functions and Their Graphs

Increasing, Decreasing, and Constant Functions

1. A function is **increasing** on an open interval, I, if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ for any x_1 and x_2 in the interval.
2. A function is **decreasing** on an open interval, I, if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ for any x_1 and x_2 in the interval.
3. A function is **constant** on an open interval, I, if $f(x_1) = f(x_2)$ for any x_1 and x_2 in the interval.

Example: Intervals on Which a Function Increases, Decreases, or is Constant

State the intervals on which the given function is increasing, decreasing, or constant.

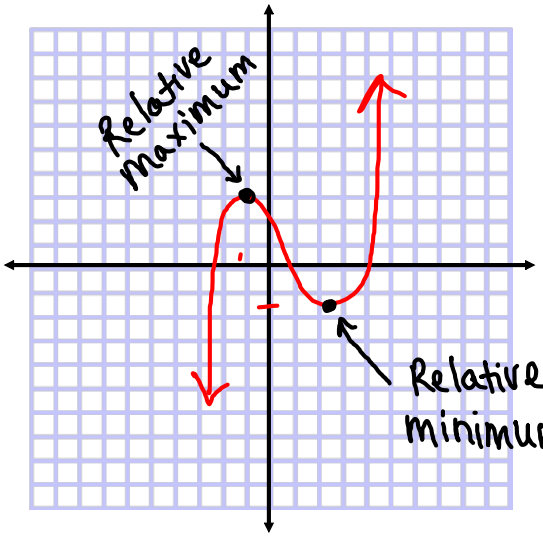


↑ : $(-\infty, -1] \cup [1, \infty)$

↓ : $[-1, 1]$

Relative Extrema

may or may not be the greatest or least value of f on the domain but it's the greatest or least y-value on some interval of the domain.

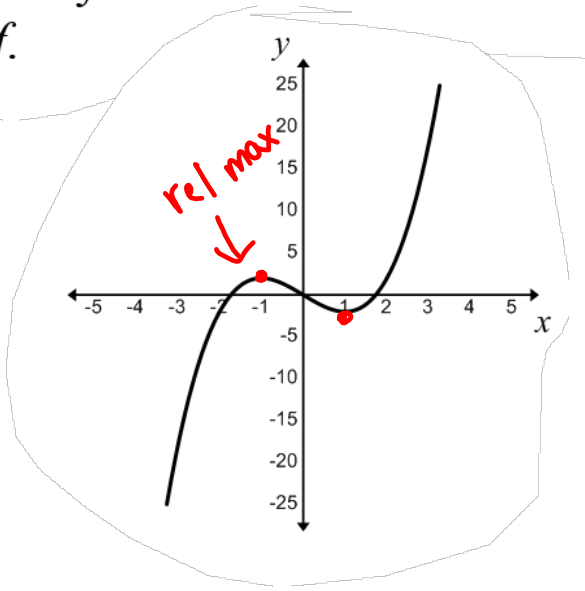


rel max @ x=-1
rel max = 3

rel min @ x=2
rel min = -1.5

Note: the x-value is where the graph has a relative max or min. The y-value is the VALUE of the relative max or min.

Identify the relative maxima and minima for the graph of f.



rel max @ x = -1
rel max = 2.5

rel min @ x = 1
rel min = -2.5

Definitions of Even and Odd Functions

The function f is an **even function** if $f(-x) = f(x)$ for all x in the domain of f. The right side of the equation of an even function does not change if x is replaced with $-x$.

$f(x) = x^2$

$(-x)^2$

x^2

The function f is an **odd function** if $f(-x) = -f(x)$ for all x in the domain of f. Every term on the right side of the equation of an odd function changes its sign if x is replaced with $-x$.

$f(x) = x^3$

$(-x)^3$

$-x^3$

Example: Identifying Even or Odd Functions

Determine whether the function $h(x) = x^5 + 1$ is even, odd, or neither.

$h(-x) = (-x)^5 + 1$

$(-1)^5 x^5$

$-h(x) = -x^5 - 1$

$-(x^5 + 1)$

$h(-x) = -x^5 + 1$

neither

Determine whether $f(x) = 3x^3 - 5x^7$ is even or odd or neither.

$f(-x) = 3(-x)^3 - 5(-x)^7$

$-3x^3 + 5x^7$

Odd

$g(x) = x^4 - 2x^2$

$g(-x) = (-x)^4 - 2(-x)^2$

$x^4 - 2x^2$

Even

Piecewise Functions

A function that is defined by two (or more) equations over a specified domain is called a **piecewise function**.

Evaluating a Piecewise Function

Given the function

$$C(t) = \begin{cases} 20 & \text{if } 0 \le t \le 60 \\ 20 + 0.40(t - 60) & \text{if } t > 60 \end{cases}$$

40 ← 20 if $0 \le t \le 60$

80 → $20 + 0.40(t - 60)$ if $t \ge 60$ ← 80

Find $C(40)$ = 20

Find $C(80)$ = 28

$20 + .4(80 - 60)$

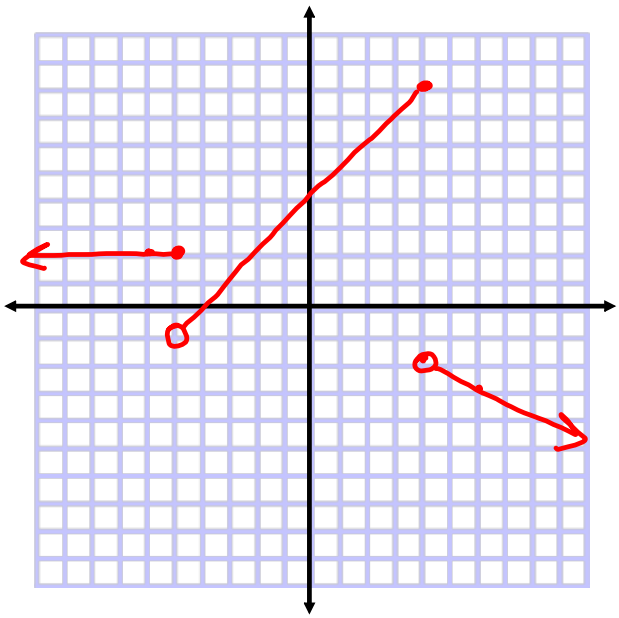
$20 + .4(20)$

$20 + 8 = 28$

When graphing piecewise functions:

1. graph constant functions first
2. remember that < or > means an open dot and $\leq$ and $\geq$ means a closed dot

$$f(x) = \begin{cases} 2 & x \leq -5 \\ x+4 & -5 < x \leq 4 \\ -\frac{1}{2}x & x > 4 \end{cases}$$

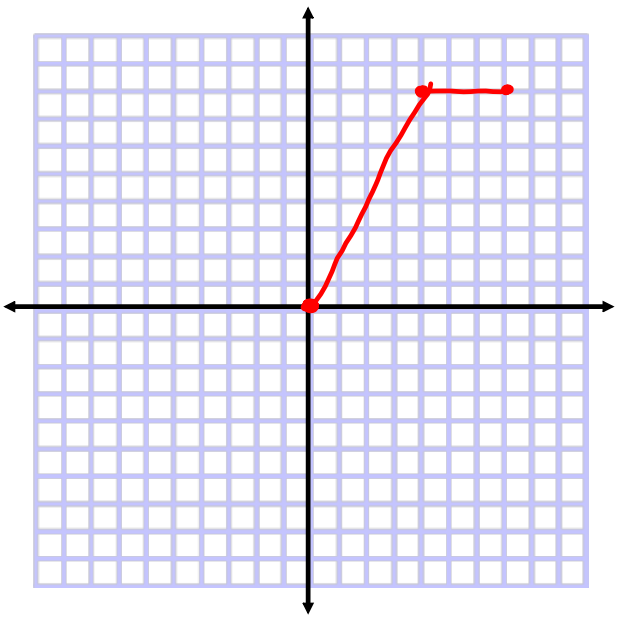


• (-5, 2) • (-6, 2)
○ (-5, -1) • (4, 8)
○ (4, -2) • (6, -3)

Try this:

graph $f(x) = \begin{cases} 2x & 0 \leq x \leq 4 \\ 8 & 4 < x \leq 7 \end{cases}$

• (0, 0) • (4, 8)
○ (4, 8) • (7, 8)



Homework

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