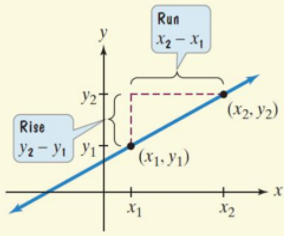
Math Analysis
1.4 & 1.5 Linear Functions and Slope

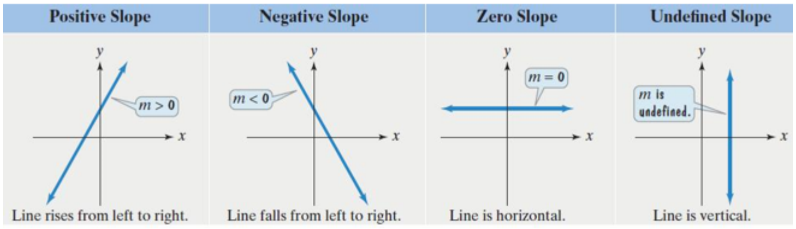
The **slope** of the line through the distinct points (x_1, y_1) and (x_2, y_2) is

$$\frac{\text{Change in } y}{\text{Change in } x} = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1},$$

Rise: Vertical change
Run: Horizontal change

where $x_2 - x_1 \neq 0$.





Find the slope of the line passing through the points (4, –2) and (–1, 5)
$x_1 y_1$ $x_2 y_2$

$$\frac{y_2 - y_1}{x_2 - x_1} \qquad \frac{5 - -2}{-1 - 4} \qquad \frac{7}{-5}$$

A Summary of the Various Forms of Equations of Lines

1. Point-slope form	$y - y_1 = m(x - x_1)$
2. Slope-intercept form	$y = mx + b$ or $f(x) = mx + b$
3. Horizontal line	$y = b$
4. Vertical line	$x = a$
5. General form	$Ax + By + C = 0$

$(2, 4)$ $m = 4$
$y - 4 = 4(x - 2)$
$Ax + By = C$

Write an equation in point-slope form for the line with slope 6 that passes through the point (2, –5). Then solve the equation for *y*.

$m = 6$ $\quad (2, -5)$ (x_1, y_1)
$y - y_1 = m(x - x_1)$
$y - (-5) = 6(x - 2)$
$\boxed{y + 5 = 6(x - 2)}$
$y + 5 = 6x - 12$
$\quad -5 \qquad -5$
$\boxed{y = 6x - 17}$

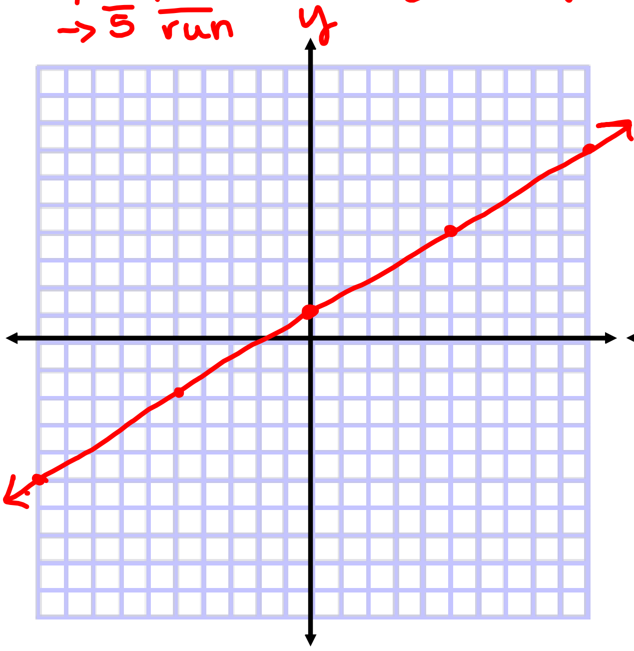
Write an equation in slope-intercept form that passes through the points (4, -3) and (-2, 6).
$m = -3/2$
$b = 3$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 + 3}{-2 - 4} = \frac{9}{-6}$
$m = -\frac{3}{2}$

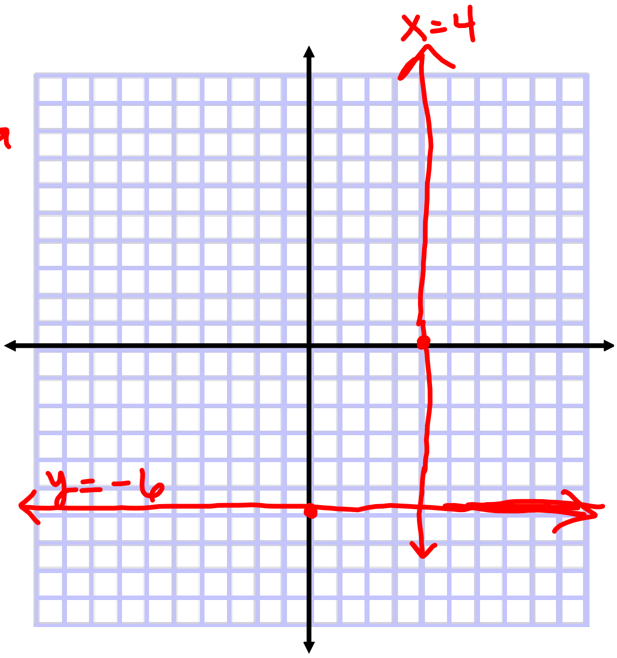
$y = mx + b$
$6 = -\frac{3}{2}(-2) + b$
$6 = 3 + b$
$-3 \quad -3$
$3 = b$
$\boxed{y = -\frac{3}{2}x + 3}$

Graph the linear function: $f(x) = \frac{3}{5}x + 1$. y b y-int

$\uparrow \frac{3}{5} \frac{\text{rise}}{\text{run}} \rightarrow$



Graph x = 4 and y = -6



Find the slope and y-intercept of the line whose equation is 3x + 2y - 4 = 0.
Then graph the equation.

$-3x \quad +4 \quad -3x \; +4$

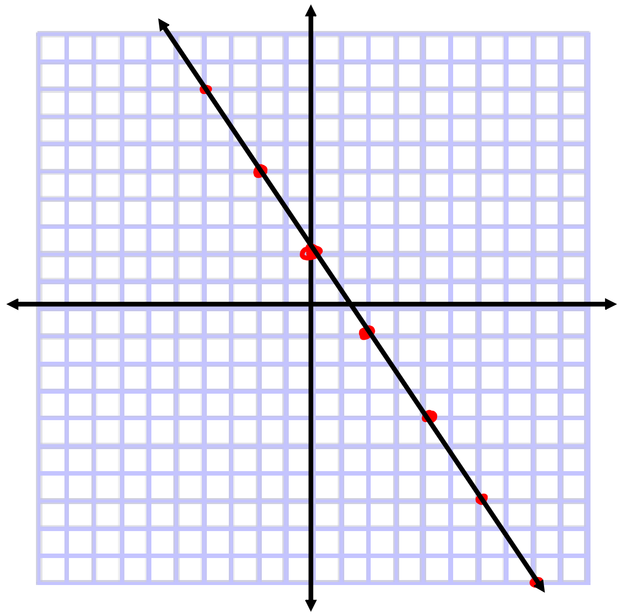
$y = mx + b$

$\frac{2y}{2} = -\frac{3x}{2} + \frac{4}{2}$

$y = -\frac{3}{2}x + 2$

$\boxed{m = -\frac{3}{2} \quad b = 2}$

$\frac{3}{-2}$



Find the x & y-intercepts and then use them to graph the equation.

$$4x - 3y - 6 = 0$$

y-int: $x=0$
$(0,-2)$

y-int let $x=0$ & solve for y

$4(0) - 3y - 6 = 0$

$-3y - 6 = 0$

$\frac{-3y}{-3} = \frac{6}{-3}$

$y = -2$

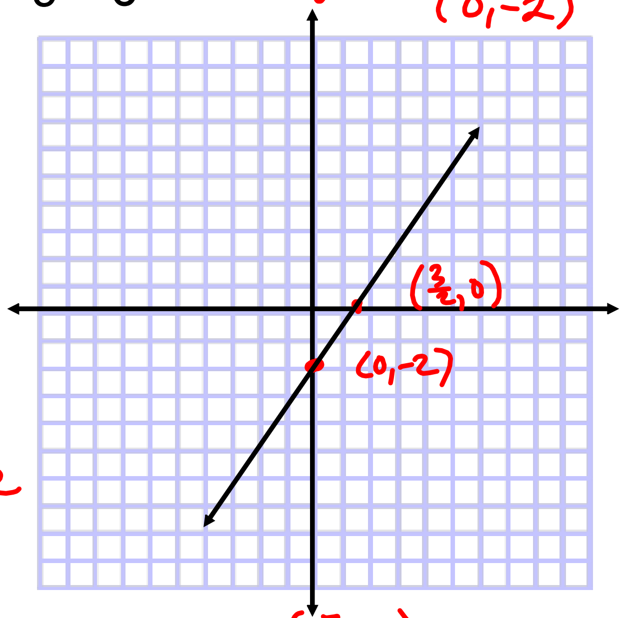
x-int: $y=0$ & solve for x

$4x - 3(0) - 6 = 0$

$4x - 6 = 0$

$4x = 6$

$x = 6/4 = 3/2$



x-int: $(\frac{3}{2}, 0)$

Parallel Lines have the SAME slope

Perpendicular Lines have OPPOSITE RECIPROCAL slopes

Write an equation in point-slope form that goes through (-3, 1) and is parallel to the line y = 2x + ~~1~~

$m = 2$ $(-3, 1)$ x_1, y_1

$y - y_1 = m(x - x_1)$

$$\boxed{y - 1 = 2(x + 3)}$$

Write an equation in slope-intercept form that goes through the point (3, -5) and is perpendicular to the line x + 4y - 8 = 0

$y = mx + b$

$-x$ $+8$ $-x+8$

$\perp$ opp recip

x y
$(3, -5)$

$m = +\frac{4}{1} = 4$

$\frac{4y}{4} = \frac{-x}{4} + \frac{8}{4}$

$y = -\frac{1}{4}x + \cancel{2}$ m

$y = mx + b$

$-5 = 4(3) + b$

$-5 = 12 + b$
$-12 \quad -12$

$-17 = b$

$$\boxed{y = 4x - 17}$$

Example: Slope as Rate of Change

In 1990, 9 million adult men in the United States lived alone. In 2008, 14.7 million adult men in the United States lived alone. Use this information to find the slope of the linear function representing adult men living alone in the United States. Express the slope correct to two decimal places and describe what it represents.

(1990, 9)

(2008, 14.7)

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{14.7 - 9}{2008 - 1990} = \frac{5.7}{18}$$

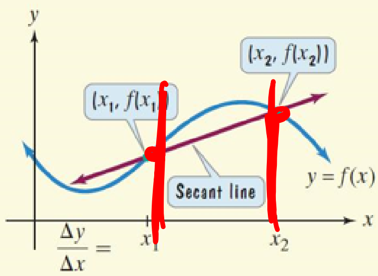
$\boxed{.32}$

increasing the # of adult men by .32 million/yr

The Average Rate of Change of a Function

Let $(x_1, f(x_1))$ and $(x_2, f(x_2))$ be distinct points on the graph of a function f. (See **Figure 1.52**.) The **average rate of change of f** from x_1 to x_2, denoted by $\frac{\Delta y}{\Delta x}$ (read "delta y divided by delta x" or "change in y devided by change in x"), is

$$\frac{\Delta y}{\Delta x} = \frac{\overset{y}{f(x_2)} - \overset{y}{f(x_1)}}{x_2 - x_1}.$$



Find the average rate of change for $f(x) = x^3$ from $x_1 = -2$ to $x_2 = 0$.

$(-2)^3$

0^3

$(-2, -8)(0, 0)$

$$\frac{0 - {}^-8}{0 - {}^-2} = \frac{8}{2} = \boxed{4}$$

Example: Finding Average Velocity

The distance, $s(t)$, in feet, traveled by a ball rolling down a ramp is given by the function

$$s(t) = 4t^2$$

$4(1)^2 = 4$ $4(2)^2 = 16$

where t is the time, in seconds, after the ball is released. Find the ball's average velocity from $t_1 = 1$ second to $t_2 = 2$ seconds.

$(1, 4)$ $(2, 16)$

$$\frac{16-4}{2-1} = \frac{12}{1} = \boxed{12 \text{ ft/s}}$$

Homework

pg 199; 26, 32, 38, 42, 44, 48-52e, 60, 62, 68, 70
pg 212; 5-8, 10-18e, 19, 20 ec: 30, 34
#10 + 12 don't write in general form