Geometry
1.4 Reasoning and Proof

A **conjecture** is a statement that is believed to be true. You can use *inductive* or *deductive* reasoning to show, or *prove*, that a conjecture is true. **Inductive reasoning** is the process of reasoning that a rule or statement is true because specific cases are true. **Deductive reasoning** is the process of using logic to draw conclusions.

A) Make a conjecture about the sum of three consecutive counting numbers.

+ right in a row

1, 2, 3, 4, ...

$1+2+3=6$
$2+3+4=9$
$3+4+5=12$
$24+25+26=75$
$33+34+35=102$

The sum of 3 consecutive counting numbers is divisible by 3.

Recall that postulates are statements you accept are true. A **theorem** is a statement that you can prove is true using a series of logical steps. The steps of deductive reasoning involve using appropriate undefined words, defined words, mathematical relationships, postulates, or other previously-proven theorems to prove that the theorem is true.

Use deductive reasoning to prove that the sum of three consecutive counting numbers is divisible by 3.

1st #		2nd #		3rd #		
x	+	$x+1$	+	$x+2$	=	$3x+3$

$3(x+1)$

So yes the sum of 3 consecutive counting #'s is divisible by 3.

Reflect

1. **Discussion** A **counterexample** is an example that shows a conjecture to be false. Do you think that counterexamples are used mainly in inductive reasoning or in deductive reasoning?

a counterexample would be used in inductive reasoning to show that at least one specific case makes the conjecture false.

A **conditional statement** is a statement that can be written in the form "If p, then q" where p is the *hypothesis* and q is the *conclusion*. For example, in the conditional statement "If $3x - 5 = 13$, then $x = 6$," the hypothesis is "$3x - 5 = 13$" and the conclusion is "$x = 6$."

Properties of Equality	
Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $ac = bc$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Reflexive Property of Equality	$a = a$
Symmetric Property of Equality	If $a = b$, then $b = a$.
Transitive Property of Equality	If $a = b$ and $b = c$, then $a = c$.
Substitution Property of Equality	If $a = b$, then b can be substituted for a in any expression.

Example 1 **Use deductive reasoning to solve the equation. Use the Properties of Equality to justify each step.**

(A) $14 = 3x - 4$

$+4 \quad +4$

$\frac{18}{3} = \frac{3x}{3}$ Add Prop.

$6 = x$ Div Prop

(B) $9 = 17 - 4x$

$-17 \quad -17$

$\frac{-8}{-4} = \frac{-4x}{-4}$ Subtract Prop

$2 = x$ Div Prop

Your Turn

5. Use deductive reasoning to solve the equation $3 - 4x = -5$.

$-3 \quad -3$ Subtract Prop

$-4x = -8$

$\overline{-4} \quad \overline{-4}$

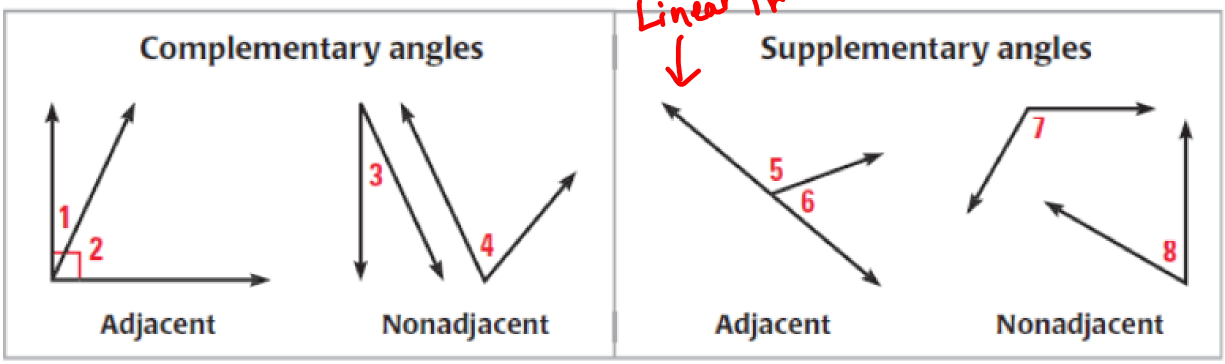
$x = 2$ Div Prop

6. Identify the Property of Equality that is used in each statement.

If $x = 2$, then $2x = 4$.	Mult. Prop.
$5 = 3a$; therefore, $3a = 5$.	Symmetric
If $T = 4$, then $5T + 7$ equals 27.	Substitution
If $9 = 4x$ and $4x = m$, then $9 = m$.	Transitive

complementary angles: 2 angles whose measures add to 90°
supplementary angles: 2 angles whose measures add to 180°
adjacent angles: 2 angles that share a common vertex and side but have no common interior points.

Linear Pr

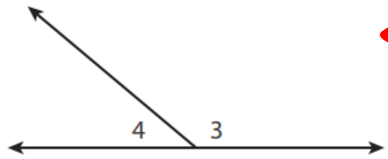


Recall that two angles whose measures add up to 180° are called *supplementary angles*. The following theorem shows one type of supplementary angle pair, called a *linear pair*. A **linear pair** is a pair of adjacent angles whose non-common sides are opposite rays. You will prove this theorem in an exercise in this lesson.

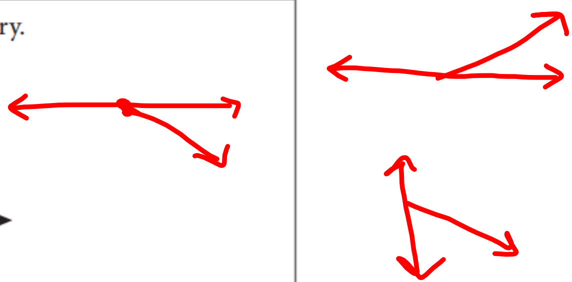
<u>opposite rays</u>: rays that share a common endpoint and form a line.

The Linear Pair Theorem

If two angles form a linear pair, then they are supplementary.

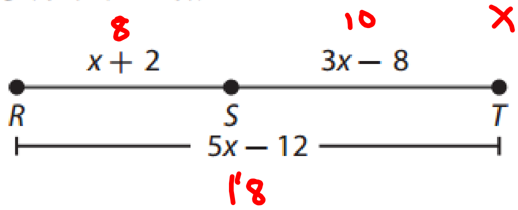


$m\angle 3 + m\angle 4 = 180°$



Example 2 Use a postulate or theorem to find the value of x in each figure.

(A) Given: $RT = 5x - 12$



Seg. Add Prop.

$RS + ST = RT$

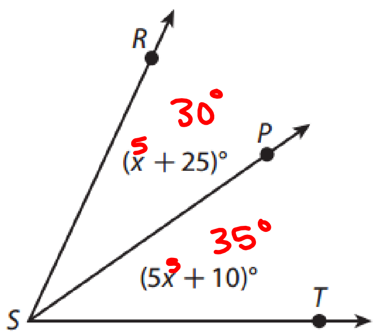
$x+2 + 3x-8 = 5x-12$

$4x - 6 = 5x - 12$

$-6 = x - 12$

$6 = x$

(B) Given: $m\angle RST = (15x - 10)°$ 65°



∡ Add Prop.

$m\angle RSP + m\angle PST = m\angle RST$

$x+25 + 5x+10 = 15x-10$

$6x+35 = 15x-10$

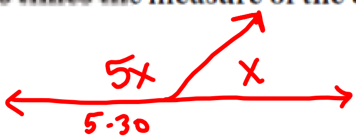
$35 = 9x-10$

$45 = 9x$

$5 = x$

EXAMPLE 5 **Find angle measures in a linear pair**

ALGEBRA Two angles form a linear pair. The measure of one angle is 5 times the measure of the other. Find the measure of each angle. =180°



$5x + x = 180$

$6x = 180$

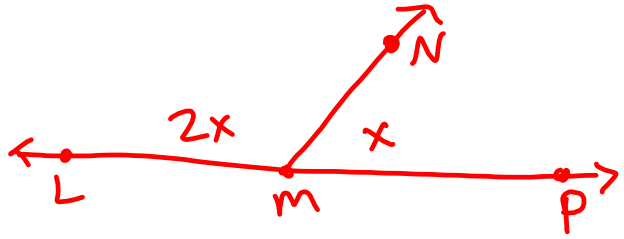
$x = 30$

150° + 30°

Your Turn

8. Two angles *LMN* and *NMP* form a linear pair. The measure of ∠*LMN* is twice the measure of ∠*NMP*. Find m∠*LMN*.

= 180



$x + 2x = 180$
$3x = 180$
$x = 60$

$m\angle LMN = 2x = 2 \cdot 60 = 120^\circ$

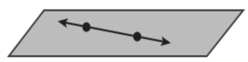
Postulates about Points, Lines, and Planes

Through any two points, there is exactly one line.

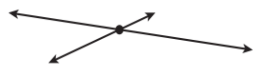
Through any three noncollinear points, there is exactly one plane containing them.

pg 43 in book

If two points lie in a plane, then the line containing those points lies in the plane.

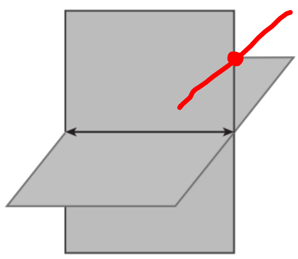


If two lines intersect, then they intersect in exactly one point.



pts lines planes

If two planes intersect, then they intersect in exactly one line.

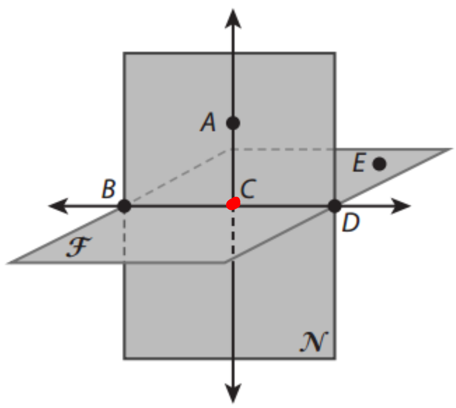


line + plane

line
or
pt

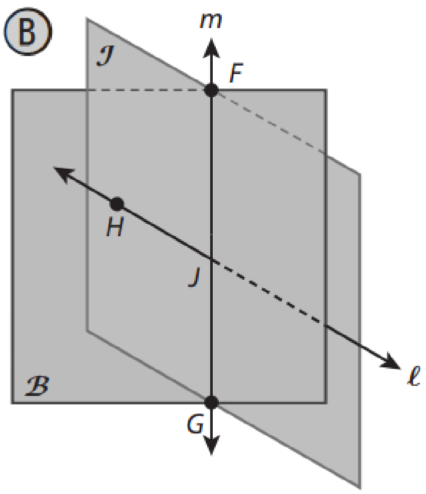
Example 3 Use each figure to name the results described.

(A)



Description
the line of intersection of two planes $\overleftrightarrow{BD}$
the point of intersection of two lines C or B or D
three coplanar points ACD ABD ABC BCE ECD
three collinear points BCD

(B)



Description	Example from the figure
the line of intersection of two planes	$\overleftrightarrow{FG}$
the point of intersection of two lines	J
three coplanar points	HJF
three collinear points	FJG

Homework

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