

Math Analysis
1.9 Circles



The Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Find the distance between $(-1, -3)$ and $(2, 3)$. Express the answer in simplified radical form and then round to two decimal places.

$$\begin{aligned}
 d &= \sqrt{(2+1)^2 + (3+3)^2} \\
 &= \sqrt{3^2 + 6^2} \\
 &= \sqrt{9 + 36} \\
 &= \sqrt{45} \\
 &= \sqrt{9 \cdot 5} \\
 &= 3\sqrt{5} \approx 6.71
 \end{aligned}$$

The Midpoint Formula

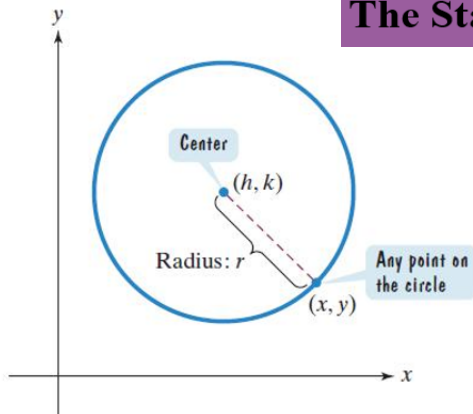
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Find the midpoint of the line segment with endpoints $(1, 2)$ and $(7, -3)$.

$$\left(\frac{1+7}{2}, \frac{2+(-3)}{2} \right) = \left(4, -\frac{1}{2} \right)$$

A **circle** is the set of all points in a plane that are equidistant from a fixed point, called the **center**. The fixed distance from the circle's center to any point on the circle is called the **radius**.

The Standard Form of the Equation of a Circle



$$(x - h)^2 + (y - k)^2 = r^2$$

Example: Finding the Standard Form of a Circle's Equation

- A) Write the standard form of the equation of the circle with center $(0, -6)$ and radius 10.

$$(x - 0)^2 + (y - -6)^2 = 10^2$$

$$x^2 + (y + 6)^2 = 100$$

- B) Center $(-7, 5)$ radius 4

$$(x - -7)^2 + (y - 5)^2 = 4^2$$

$$(x + 7)^2 + (y - 5)^2 = 16$$

Example: Using the Standard Form of a Circle's Equation to Graph the Circle

Find the center and the radius of the circle. Graph. Then use the graph of the equation to identify the relation's domain and range.

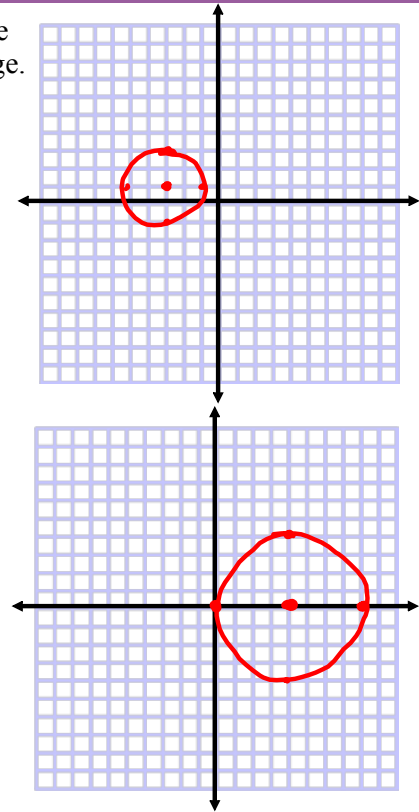
A) $(x+3)^2 + (y-1)^2 = 4$

$C(h,k) = (-3, 1)$

$r = 2$

B) $(x-4)^2 + y^2 = 16$

$(4, 0)$ $r = 4$



Try this:

1) Write the standard form of the equation of the circle with center $(-2, -6)$ and radius 12.

$(x+2)^2 + (y+6)^2 = 144$

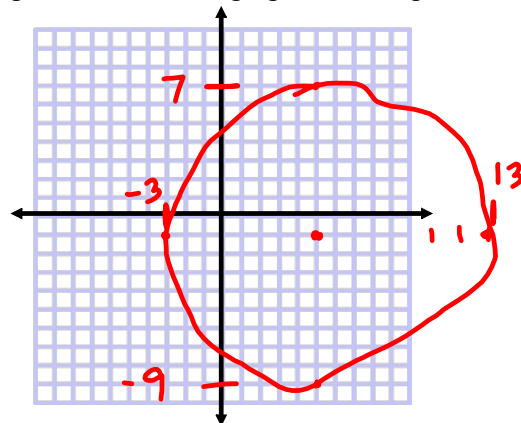
2) Find the center and the radius of the circle. Graph. Then use the graph of the equation to identify the relation's domain and range.

$(x-5)^2 + (y+1)^2 = 64$

$C(5, -1)$ $r = 8$

$D: [-3, 13]$

$R: [-9, 7]$



The General Form of the Equation of a Circle

The general form of the equation of a circle is

$$x^2 + y^2 + Dx + Ey + F = 0$$

where D , E , and F are real numbers.

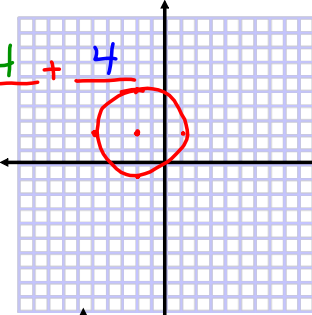
Write the standard form of the equation. Find the radius and the center. Graph.

A) $x^2 + y^2 + 4x - 4y - 1 = 0$

$$\underbrace{x^2 + 4x + 4}_{(x+2)^2} + \underbrace{y^2 - 4y + 4}_{(y-2)^2} = 1 + 4 + 4$$

$$(x+2)^2 + (y-2)^2 = 9$$

C: $(-2, 2)$ $r = 3$



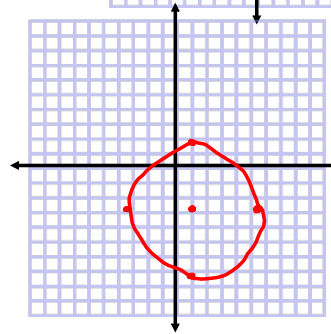
B) $2x^2 + 2y^2 - 4x + 12y - 18 = 0$

$$x^2 + y^2 - 2x + 6y - 9 = 0$$

$$x^2 - 2x + 1 + y^2 + 6y + 9 = 9 + 1 + 9$$

$$(x-1)^2 + (y+3)^2 = 19$$

C: $(1, -3)$ $r = \sqrt{19} \approx 4.4$



Try this: The equation of a circle is $4x^2 + 4y^2 - 24x + 16y = -51$.

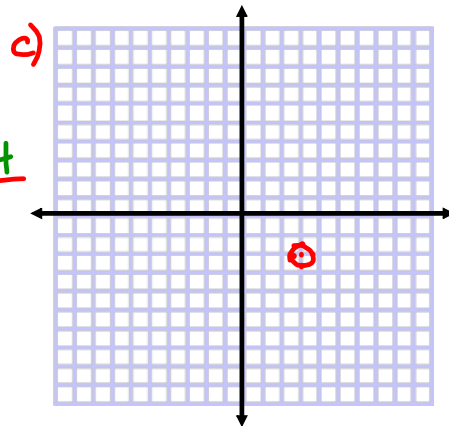
- Write the equation in standard form.
- Find the radius and the coordinates of the center.
- Graph the equation.

$$x^2 + y^2 - 6x + 4y = \frac{-51}{4}$$

$$x^2 - \left(\frac{6}{2}\right)x + \frac{9}{4} + y^2 + \left(\frac{4}{2}\right)y + 4 = \frac{-51}{4} + \frac{9}{4} + \frac{4}{4}$$

$$a) (x-3)^2 + (y+2)^2 = \frac{1}{4}$$

b) C: $(3, -2)$ $r = \frac{1}{2}$



Homework:

pg. 264; 4, 24, 34-44e, 50-60e
ec: 72, 74