

Math Analysis
2.1 Complex Numbers

$$i^2 = -1$$

Comes from

$$i^2 = -1$$

$$\sqrt{i^2} = \sqrt{-1}$$

$$i = \sqrt{-1}$$

Standard Form of a Complex number: $a + bi$

Note: $3 + (-2)i = 3 - 2i$
 $a + 0i = a$
 $0 + bi = bi$

real part imaginary part

Add, Subtract, Multiply, Divide Complex Numbers

Ex 1) a) $(3+5i) + (-2+3i)$
 $3 + -2 \quad 5i + 3i$
 $1 + 8i$

b) $(6+4i) - (3+6i)$
 $6 + 4i - 3 - 6i$
 $3 - 2i$

Ex 2) $(5+3i)(2+7i)$ Remember $i^2 = -1$

$$5 \cdot 2 \quad 5 \cdot 7i \quad 3i \cdot 2 \quad 3i \cdot 7i$$

$$10 + 35i + 6i + 21i^2$$

$$10 + 41i - 21$$

$$-11 + 41i$$

Conjugates

Notation: \bar{z} or $\overline{a+bi} = a-bi$

Ex) a) $\overline{2+3i} = 2-3i$

b) $\overline{-6-2i} = -6+2i$

Multiplying a complex number by its conjugate will = $a^2 + b^2$

Ex) If $z = 3-4i$ find $z\bar{z}$

$$(3-4i)(3+4i)$$

$$9 - 12i + 12i - 16i^2$$

cancel

$$9 - 16(-1)$$

$$9 + 16 = 25$$

Writing the quotient of 2 complex numbers in standard form

Ex 5) a) $\frac{1+4i}{5-2i} \cdot \frac{5+2i}{5+2i}$ like rationalizing the denominator

$$\frac{(1+4i)(5+2i)}{(5-2i)(5+2i)} = \frac{5+2i+20i+8i^2}{25+4}$$

$$\frac{-3+22i}{29} = \frac{-3}{29} + \frac{22i}{29}$$

b) $\frac{2-3i}{4-3i} \cdot \frac{4+3i}{4+3i} = \frac{8+6i-12i-9i^2}{4^2+3^2}$

$$= \frac{17-6i}{25} = \frac{17}{25} - \frac{6i}{25}$$

Try this:

Perform the indicated operations, writing the result in standard form:

a. $(5-2i) + (3+3i)$
 $8+i$

b. $(2+6i) - (12-i)$
 $-10+7i$

c. $7i(2-9i)$

$$14i - 63i^2$$

$$63+14i$$

d. $(5+4i)(6-7i)$

$$30 + 24i - 35i + 28i^2$$

$$-28i^2 = -28(-1) = 28$$

$$58-11i$$

e) $\frac{5+4i}{4-i} \cdot \frac{4+i}{4+i}$

$$\frac{20+16i+5i-4}{16+1}$$

$$= \frac{16+21i}{17}$$

$$= \frac{16}{17} + \frac{21}{17}i$$

Evaluating the Square Root of a Negative Number

$$\sqrt{-b} = i\sqrt{b}$$

a) $\sqrt{-1}$
 $i\sqrt{1}$
 $i \cdot 1$
 i

b) $\sqrt{-4}$
 $i\sqrt{4}$
 $i \cdot 2$
 $2i$

c) $\sqrt{-8}$
 $i\sqrt{8}$
 $2i\sqrt{2}$

Perform the indicated operations and write the result in standard form.

a. $\sqrt{-27} + \sqrt{-48}$

$$i\sqrt{27} + i\sqrt{48}$$

$$3i\sqrt{3} + 4i\sqrt{3}$$

$$(3i+4i)\sqrt{3}$$

$$7i\sqrt{3}$$

b. $(-2+\sqrt{-3})^2$

$$(-2+\sqrt{-3})(-2+\sqrt{-3})$$

FOIL

$$4 - 2\sqrt{-3} - 2\sqrt{-3} + (\sqrt{-3})^2$$

$$4 - 4\sqrt{-3} - 3$$

$$1 - 4i\sqrt{3}$$

Solve equations with complex solutions

$$\sqrt{x^2} = \sqrt{-9}$$

$$\boxed{x = \pm 3i}$$

$$x^2 - 4x + 8 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{4 \pm \sqrt{16 - 4(1)(8)}}{2}$$

$$\frac{4 \pm \sqrt{-16}}{2} = \frac{4 \pm 4i}{2}$$

$$\boxed{2 \pm 2i}$$

Homework

pg 298; 2,4,10-16e, 22-34e, 38-50e

ec:70, 82