Math Analysis
2.3 part 1; Polynomial Functions & Their Graphs
(Definition of Polynomial and End Behavior)

Definition of a Polynomial Function

Let n be a nonnegative integer and let $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$ be real numbers, with $a_n \neq 0$. The function defined by

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

is called a **polynomial function of degree *n***. The number a_n, the coefficient of the variable to the highest power, is called the **leading coefficient**.

Polynomial Fns	Not Polynomial Fns
$f(x) = -3x^5 + \sqrt{2}x^2 + 5$	$F(x) = -3\sqrt{x} + \sqrt{2}x^2 + 5$
polynomial degree 5	$= -3x^{\frac{1}{2}} + \sqrt{2}x^2 + 5$
	The exponent on the variable is not an integer
$g(x) = -3x^4(x-2)(x+3)$	$G(x) = -\frac{3}{x^2} + \sqrt{2}x^2 + 5$
polynomial degree 6	$= -3x^{-2} + \sqrt{2}x^2 + 5$
	The exponent on the variable is not a nonnegative integer

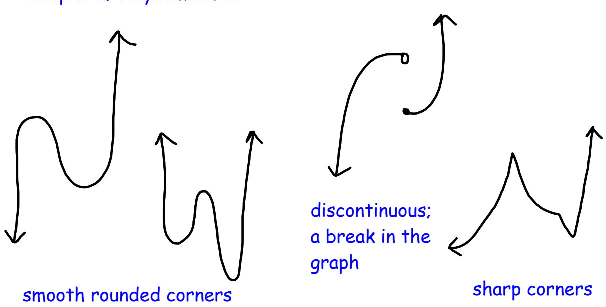
Polynomial functions of degree 2 or higher have graphs that are *smooth* and *continuous*.

By **smooth**, we mean that the graphs contain only rounded curves with no sharp corners.

By **continuous**, we mean that the graphs have no breaks and can be drawn without lifting your pencil from the rectangular coordinate system.

Graphs of Polynomial Fns

Not Graphs of Polynomial Fns



End Behavior: describes what the y-values do as |x| gets larger and larger

$x \to \infty$: As x approaches infinity

$x \to -\infty$: " " " negative infinity

Describing End Behavior

Use calculator, make a table of values

Describe the end behavior of $g(x) = -5x^3 + 4x^2 - 2x + 4$

x	y
-10,000	5,000,400,020,004
-1,000	5,004,00 30,004
-100	5040204
-10	5424
0	4
10	-4616
100	-4960196
1,000	-4996001996
10,000	-4.99×10^{12}

$-\infty$ ... ∞

As $x \to \infty$, $f(x) \to -\infty$

As $x \to -\infty$, $f(x) \to \infty$

The sign of the leading coefficient, a_n, and the degree, n, of the polynomial function reveal its end behavior.

The Leading Coefficient Test

1. For n odd:

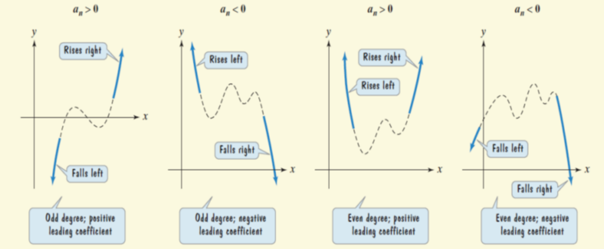
If the leading coefficient is positive, the graph falls to the left and rises to the right. (↙, ↗)

If the leading coefficient is negative, the graph rises to the left and falls to the right. (↖, ↘)

2. For n even:

If the leading coefficient is positive, the graph rises to the left and rises to the right. (↖, ↗)

If the leading coefficient is negative, the graph falls to the left and falls to the right. (↙, ↘)



Use the Leading Coefficient Test to determine the end behavior of the graph

$f(x) = 1x^{4} - 4x^2$

LC: positive

Degree (n): 4 even

As $x \to \infty, f(x) \to \infty$

As $x \to -\infty, f(x) \to \infty$

$f(x) = -4x^3(x-1)^2(x+5)^1$

3 + 2 + 1

D: 6 even

LC: neg

As $x \to \infty, f(x) \to -\infty$

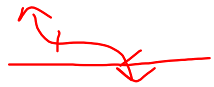
As $x \to -\infty, f(x) \to -\infty$

Use end behavior to explain why this function is only an appropriate model for AIDS diagnoses for a limited time period.

$f(x) = -49x^3 + 806x^2 + 3776x + 2503$

LC: neg

D: 3 odd



yes, for a limited time only b/c it eventually goes negative

Try This:

State the End Behavior

$f(x) = 2x^3(x-1)(x+5)$

LC: pos

D: 5 odd

As $x \to \infty, f(x) \to \infty$

As $x \to -\infty, f(x) \to -\infty$

The function below models the ratio of students to computers in U.S. public schools x years after 1980. Determine whether this function could be an appropriate model for computers in the classroom well into the twenty-first century. Explain.

$f(x) = -.27x^3 + 9.2x^2 - 102.9x + 400$

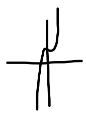
neg odd

No its not an appropriate model b/c the ratio can't go neg.

Get out a Graphing Calculator

The graph of the equation below was obtained with a graphing utility using a [-8,8,1] by [-10,10,1] viewing rectangle. Is this a complete graph that shows the end behavior of the function?

$f(x) = -x^4 + 8x^3 + 4x^2 + 2$



LC: neg D = even

As $x \to \infty, f(x) \to -\infty$

As $x \to -\infty, f(x) \to -\infty$

NO

Now change the viewing rectangle to [-10,10,1] by [-1000,750,250]

Homework

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