Math Analysis

2.4 Polynomial Long Division, Synthetic Division, Factor Theorem

When you divide a polynomial $f(x)$ by a divisor $d(x)$, you get a quotient polynomial $q(x)$ and a remainder polynomial $r(x)$.

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

The degree of the remainder must be less than the degree of the divisor. One way to divide polynomials is called **polynomial long division**.

EXAMPLE 1 **Use polynomial long division**

Divide $f(x) = 3x^4 - 5x^3 + 4x - 6$ by $x^2 - 3x + 5$.

$0x^2$

$$\boxed{3x^2 + 4x - 3 + \frac{-25x + 9}{x^2 - 3x + 5}}$$

$x^2 - 3x + 5 \,\overline{)\, 3x^4 - 5x^3 + 0x^2 + 4x - 6}$

$-(3x^4 - 9x^3 + 15x^2)$

$4x^3 - 15x^2 + 4x$

$-(4x^3 - 12x^2 + 20x)$

$-3x^2 - 16x - 6$

$-(-3x^2 + 9x - 15)$

$-25x + 9$

$\frac{3x^4}{x^2} = 3x^2$ $\quad \frac{4x^3}{x^2} = 4x$ $\quad \frac{-3x^2}{x^2} = -3$

Synthetic Division

You can use synthetic division to divide polynomials. The divisor has to be linear though to work.

linear divisors: $x - 2$, $x + 4$, etc.

Example: $(x^4 + 0x^3 - 7x^2 + 9x - 10) \div (x - 2)$

$x - 2 = 0$

$x = 2$

2	1	0	-7	9	-10
+	↓	2	4	-6	6
	1	2	-3	3	-4

Remainder

$$\boxed{1x^3 + 2x^2 - 3x + 3 + \frac{-4}{x-2}}$$

Depressed Polynomial

Divide

$(x^4 - 6x^3 + 0x^2 - 40x + 33) \div (x - 7)$

7	1	-6	0	-40	33
+	↓	7	7	49	63
	1	1	7	9	96

$$\boxed{x^3 + x^2 + 7x + 9 + \frac{96}{x-7}}$$

Factor Theorem: A polynomial f(x) has a factor x - k if and only if f(k) = 0 where f(k) is the remainder.

Examples of factors and zeros for: $x^2 + 7x + 12 = (x + 3)(x + 4)$

x + 3 is a factor, -3 is a zero

x + 4 is a factor, -4 is a zero

So if you are given a factor of a polynomial you can divide that factor out and you are left with a **depressed polynomial**.

Factor $f(x) = x^2 + 7x + 12$ completely given that $x + 4$ is a factor.

-4	1	7	12
		-4	-12
	1	3	0

$1x + 3$

$$\boxed{(x+4)(x+3)}$$

Solve $3x^3 - 4x^2 - 28x - 16 = 0$ given that $x + 2$ is a factor.

$$\begin{array}{r|rrrr} -2 & 3 & -4 & -28 & -16 \\ + & \downarrow & -6 & 20 & 16 \\ \hline & 3 & -10 & -8 & 0 \end{array}$$

$(3x^2 - 10x - 8)(x+2)$

$(3x + 2)(x - 4)(x+2)$

$3x+2=0$

$\frac{3x}{3} = \frac{-2}{3}$

$\boxed{x = -2/3, 4, -2}$

A polynomial *f* and one zero of *f* are given. Find the other zeros of *f*.

$f(x) = 1x^3 + 2x^2 - 20x + 24; -6$ zero

$$\begin{array}{r|rrrr} -6 & 1 & 2 & -20 & 24 \\ + & \downarrow & -6 & 24 & -24 \\ \hline & 1 & -4 & 4 & 0 \end{array}$$

$(x^2 - 4x + 4)(x+6)$

$(x - 2)(x - 2)(x+6)$

$x = 2$ mult. 2, -6

Try this:

A polynomial *f* and one zero of *f* are given. Find the other zeros of *f*.

$f(x) = 2x^3 + 3x^2 - 39x - 20; 4$

$$\begin{array}{r|rrrr} +4 & 2 & 3 & -39 & -20 \\ & & 8 & 44 & 20 \\ \hline & 2 & 11 & 5 & 0 \end{array}$$

F L

$2x^2 + 11x + 5$ $\frac{5}{1 \cdot 5}$

F L F L

$(2x + 1)(1x + 5)$

$-1/2, -5, 4$

Homework:

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