Math Analysis

2.5 part 1 Rational Zero Theorem

The Rational Zero Theorem

If $f(x) = a_n x^n + \cdots + a_1 x + a_0$ has *integer* coefficients, then every rational zero of f has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

EXAMPLE 1 **List possible rational zeros**

List the possible rational zeros of f using the rational zero theorem.

a. $f(x) = x^3 + 2x^2 - 11x + 12$

$\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1}$

$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

b. $f(x) = 4x^4 - x^3 - 3x^2 + 9x - 10$

$\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 5, \pm 10}{\pm 1, \pm 2, \pm 4}$

$\pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{5}{4}$

GUIDED PRACTICE for Example 1

List the possible rational zeros of f using the rational zero theorem.

1. $f(x) = x^3 + 9x^2 + 23x + 15$

$\frac{p}{q} = \frac{\pm 1, \pm 3, \pm 5, \pm 15}{\pm 1}$

$\frac{p}{q} = \pm 1, \pm 3, \pm 5, \pm 15$

2. $f(x) = 2x^3 + 3x^2 - 11x - 6$

$\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2}$

$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$

Find all real zeros of $f(x) = x^3 - 5x^2 + 7x - 35$.

$5, \pm i\sqrt{7}$

$\frac{p}{q} = \pm 1, \pm 5, \pm 7, \pm 35$

$5 \mid 1 \quad -5 \quad 7 \quad -35$

$+ \quad \downarrow \quad 5 \quad 0 \quad 35$

$1 \quad 0 \quad 7 \quad 0$ ✓

$x^2 + 7 = 0$

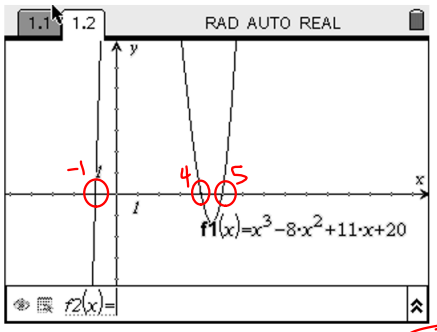
$\sqrt{x^2} = \sqrt{7}$

$x = \pm i\sqrt{7}$

EXAMPLE 2 **Find zeros**

Find all real zeros of $f(x) = x^3 - 8x^2 + 11x + 20$.

Start by Graphing the function in your calculator. Use the trace button and/or your Table to approximate the best p/q choice to try by looking at where the graph crosses the x-axis.



$-1, 4, 5$

$\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

$-1 \mid 1 \quad -8 \quad 11 \quad 20$

$\quad\quad -1 \quad 9 \quad -20$

$1 \quad -9 \quad 20 \quad 0$ ✓

$x^2 - 9x + 20$

$(x-4)(x-5)$

$x = 4, 5$

$4 \mid 1 \quad -9 \quad 20$

$\quad\quad 4 \quad -20$

$1 \quad -5 \quad 0$

$x - 5$

Find all real zeros of $f(x) = 10x^4 - 11x^3 - 42x^2 + 7x + 12$.

$\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1, \pm 2, \pm 5, \pm 10}$

$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{2}, \pm \frac{1}{5}, \pm \frac{1}{10}, \pm \frac{2}{5}, \pm \frac{3}{2}, \pm \frac{3}{5}, \pm \frac{3}{10}, \pm \frac{4}{5}, \pm \frac{6}{5}, \pm \frac{12}{5}$

$\frac{1}{2}$ | 10 -11 -42 7 12
-5 8 17 -12
10 -16 -34 24 0 ✓

$10x^3 - 16x^2 - 34x + 24$

$\frac{3}{5}$ | 10 -16 -34 24
6 -6 -24
10 -10 -40 0 ✓

$10x^2 - 10x - 40$

$10(x^2 - x - 4)$

$x = \frac{1 \pm \sqrt{1 - 4(-4)}}{2}$

$\frac{1 \pm \sqrt{17}}{2}$

$\frac{3}{5}, \frac{-1}{2}$

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