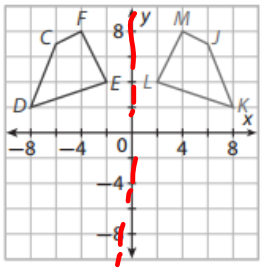
Geometry
3.2 Prove Figures Congruent using Rigid Motions

Explain 1 Determining if Figures are Congruent

Example 1 **Use the definition of congruence to decide whether the two figures are congruent. Explain your answer.**

(A)

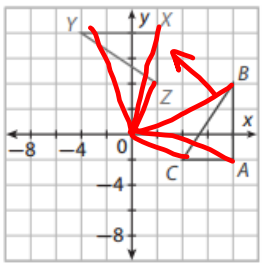


The two figures appear to be the same size and shape, so look for a rigid transformation that will map one to the other.

You can map *CDEF* onto *JKLM* by reflecting *CDEF* over the *y*-axis. This reflection is a rigid motion that maps *CDEF* to *JKLM*, so the two figures are congruent.

The coordinate notation for the reflection is $(x, y) \rightarrow (-x, y)$.

(B)



The two figures appear to be the same/different.

You can map $\triangle ABC$ to $\triangle XYZ$

by 90° rotation CCW a/b origin

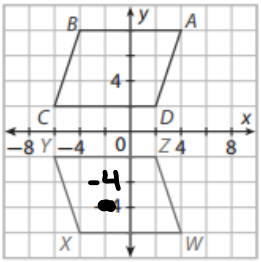
This is/is not a rigid motion that maps $\triangle ABC$ to $\triangle XYZ$, so the two figures are/are not congruent.

The coordinate notation for the rotation is _____.

Your Turn

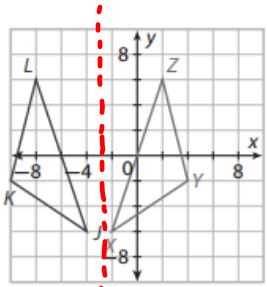
Use the definition of congruence to decide whether the two figures are congruent. Explain your answer.

2.



Reflection x-axis
Rigid
Congruent (≅)

3.



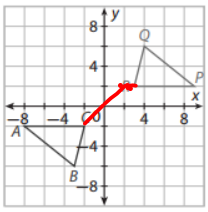
reflection x = -3
Rigid
≅

Explain 2 Finding a Sequence of Rigid Motions

The definition of congruence tells you that when two figures are known to be congruent, there must be some sequence of rigid motions that maps one to the other.

Example 2 **The figures shown are congruent. Find a sequence of rigid motions that maps one figure to the other. Give coordinate notation for the transformations you use.**

(A) $\triangle ABC \cong \triangle PQR$

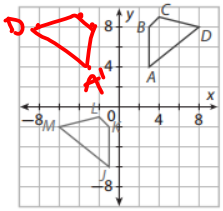


Map $\triangle ABC$ to $\triangle PQR$ with a rotation of 180° around the origin, followed by a horizontal translation.

Rotation: $(x, y) \rightarrow (-x, -y)$

Translation: $(x, y) \rightarrow (x + 1, y)$

(B) $ABCD \cong JKLM$



Map $ABCD$ to $JKLM$ with a

_____ followed by a _____

_____ : $(x, y) \rightarrow$ _____

_____ : $(x, y) \rightarrow$ _____

A(3,4) J(
A'(-3,4)

↓10 → 2

reflection y-axis (-x,y)
trans. (x+2, y-10)
<2,-10>

Reflect

4. How is the orientation of the figure affected by a sequence of transformations?

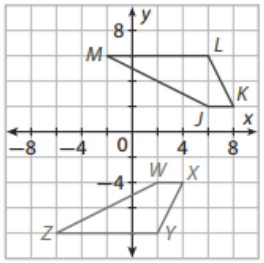
_____

_____

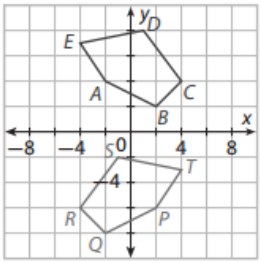
Your Turn

The figures shown are congruent. Find a sequence of rigid motions that maps one figure to the other. Give coordinate notation for the transformations you use.

5. $JKLM \cong WXYZ$



6. $ABCDE \cong PQRST$



Reflection (x,y)→(x,-y)
x-axis
Translate (x,y)→(x-4, y-2)

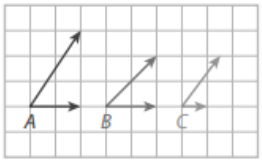
Reflect y-axis (x,y)→(-x,y)
Translate (x,y)→(x,y-10)

Explain 3 Investigating Congruent Segments and Angles

Congruence can refer to parts of figures as well as whole figures. Two angles are congruent if and only if one can be obtained from the other by rigid motions (that is, by a sequence of reflections, translations, and/or rotations.) The same conditions are required for two segments to be congruent to each other.

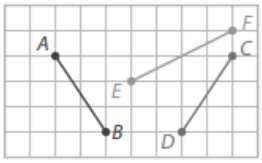
Example 3 **Determine which angles or segments are congruent. Describe transformations that can be used to verify congruence.**

A



$\angle A$ and $\angle C$ are congruent. The transformation is a translation. There is no transformation that maps $\angle B$ to either of the other angles.

B

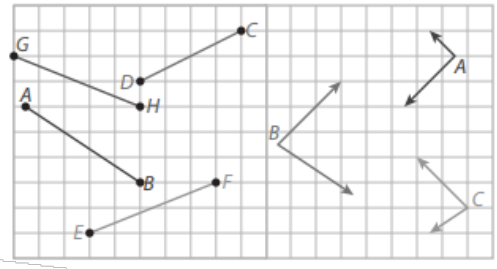


$\overline{AB}$ and $\overline{CD}$ are congruent. A sequence of transformations is a reflection and a translation.

There is no transformation that maps $\overline{EF}$ to either of the other segments.

Your Turn

7. Determine which segments and which angles are congruent. Describe transformations that can be used to show the congruence.



$\angle B \cong \angle C$

$\overline{EF} \cong \overline{GH}$

reflection then translation on both

Elaborate

8. Can you say two angles are congruent if they have the same measure but the segments that identify the rays that form the angle are different lengths?

Yes, the definition of congruence for angles requires only that the angle between the rays be the same. The length of the rays does not matter.

9. **Discussion** Can figures have congruent angles but not be congruent figures?

Yes, two figures can have congruent angles but not be congruent figures. They could appear to be different sized versions of the same figure.

10. **Essential Question Check-In** Can you use transformations to prove that two figures are not congruent?

Maybe. If a dilation with scale factor not equal to 1 maps one figure onto the other, then the figures cannot be mapped using only rigid motions, so they cannot be congruent.

Homework

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