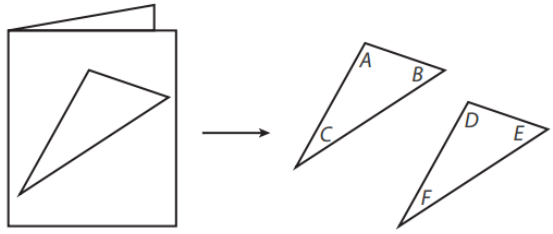
Geometry

3.3 Congruent Parts of Congruent Triangles are Congruent (CPCTC)

Fold a sheet of paper in half. Use a straightedge to draw a triangle on the folded sheet. Then cut out the triangle, cutting through both layers of paper to produce two congruent triangles. Label them $\triangle ABC$ and $\triangle DEF$, as shown.



The same sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$ maps parts of $\triangle ABC$ to parts of $\triangle DEF$. Complete the following.

$\overline{AB} \rightarrow \overline{DE}$ $\quad \overline{BC} \rightarrow \overline{EF}$ $\quad \overline{AC} \rightarrow \overline{DF}$

$\angle A \rightarrow \angle D$ $\quad \angle B \rightarrow \angle E$ $\quad \angle C \rightarrow \angle F$

What does this tell you about the corresponding parts of the two triangles? Why?

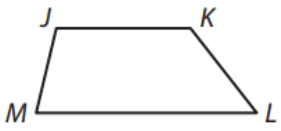
they're $\cong$ b/c they're the same size & shape

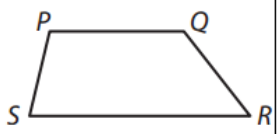
If you know that $\triangle ABC \cong \triangle DEF$, what six congruence statements about segments and angles can you write? Why?

$\angle A \cong \angle D$ $\quad \angle B \cong \angle E$ $\quad \angle C = \angle F$

$\overline{AB} \cong \overline{DE}$ $\quad \overline{BC} \cong \overline{EF}$ $\quad \overline{AC} \cong \overline{DF}$

Do your findings in this Explore apply to figures other than triangles? For instance, if you know that quadrilaterals *JKLM* and *PQRS* are congruent, can you make any conclusions about corresponding parts? Why or why not?





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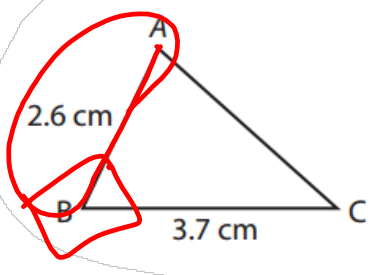
Corresponding Parts of Congruent Figures Are Congruent (CPCTC)

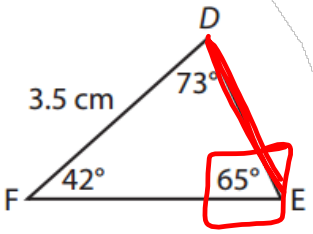
If two figures are congruent, then corresponding sides are congruent and corresponding angles are congruent.

Example 1 $\triangle ABC \cong \triangle DEF$. **Find the given side length or angle measure.**

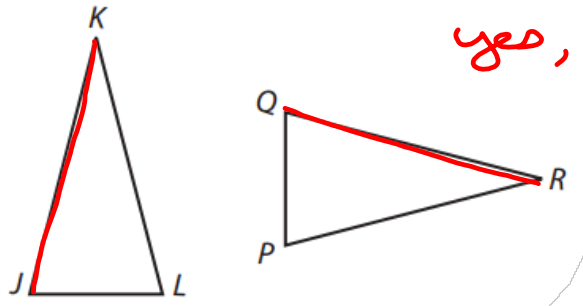
(A) $DE = AB = 2.6\text{ cm}$

(B) $m\angle B = 65°$





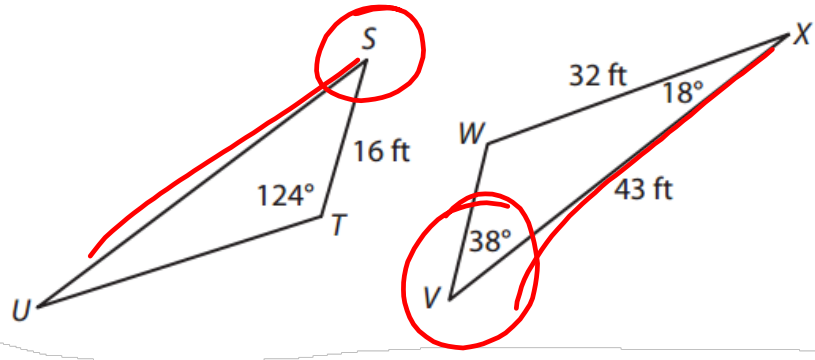
Discussion The triangles shown in the figure are congruent. Can you conclude that $\overline{JK} \cong \overline{QR}$? Explain.



yes, CPCTC

Your Turn

$\triangle STU \cong \triangle VWX$. **Find the given side length or angle measure.**



4. $SU = 43$ ft

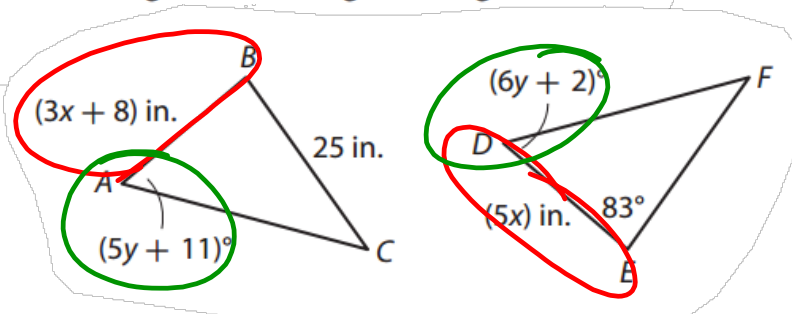
5. $m\angle S = 38°$

Rigid motions preserve length and angle measure. This means that congruent segments have the same length, so $\overline{UV} \cong \overline{XY}$ implies $UV = XY$ and vice versa. In the same way, congruent angles have the same measure, so $\angle J \cong \angle K$ implies $m\angle J = m\angle K$ and vice versa.

This is called the definition of congruence

def. of ≅

Example 2 $\triangle ABC \cong \triangle DEF$. **Find the given side length or angle measure.**



(A) $AB = 20$ in

$3x+8 = 5x$

$-3x \quad -3x$

$8 = 2x$

$x = 4$

$5 \cdot 4 = 20$

(B) $m\angle D = 56°$

$5y+11 = 6y+2$

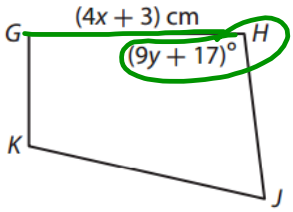
$-2 \quad -5y \quad -2$

$6 \cdot 9 + 2$

$54+2 \quad 9 = y$

Your Turn

Quadrilateral *GHJK* ≅ quadrilateral *LMNP*. Find the given side length or angle measure.

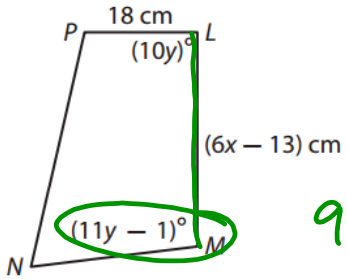


$6x-13 = 4x+3$

$2x = 16$

$x = 8 \quad 4 \cdot 8 + 3$

6. $LM = 35$ cm



7. $m\angle H = 98°$

$9y + 17 = 11y - 1$

$18 = 2y$

$y = 9$

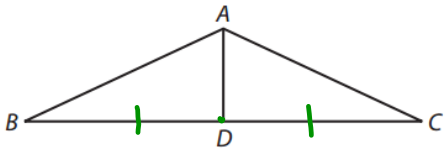
Properties of Congruence	
Reflexive Property of Congruence	$\overline{AB} \cong \overline{AB}$
Symmetric Property of Congruence	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
Transitive Property of Congruence	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Explain 3 Using Congruent Corresponding Parts in a Proof

Example 3 Write each proof.

(A) Given: $\triangle ABD \cong \triangle ACD$

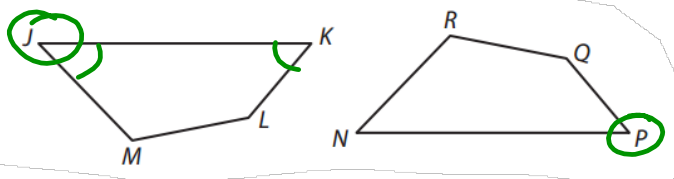
Prove: D is the midpoint of $\overline{BC}$.



Statements	Reasons
① $\triangle ABD \cong \triangle ACD$	① Given
② $\overline{BD} \cong \overline{CD}$	② Def of $\cong$ figures (CPCTC)
③ $BD = CD$	③ Def of $\cong$
④ pt D is the midpt of $\overline{BC}$	④ Def of midpoint

(B) Given: Quadrilateral $JKLM \cong$ quadrilateral $NPQR$; $\angle J \cong \angle K$

Prove: $\angle J \cong \angle P$

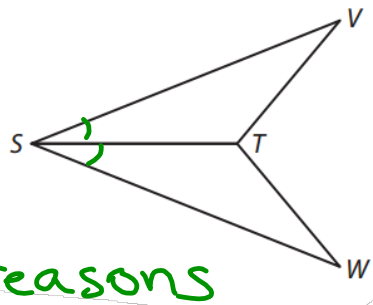


Statements	reasons
① Quad JKLM $\cong$ Quad NPQR $\angle J \cong \angle K$	① Given
~~② $\angle J = \angle N$~~	~~② Def of $\cong$ figures~~
③ $\angle K \cong \angle P$	③ Def of $\cong$ figures
④ $\angle J \cong \angle P$	④ Transitive Prop.

Your Turn

Write each proof.

8. Given: $\triangle SVT \cong \triangle SWT$
Prove: $\overline{ST}$ bisects $\angle VSW$.



Statements	Reasons
① $\triangle SVT \cong \triangle SWT$	① Given
② $\angle VST \cong \angle WST$	② CPCTC
③ ST bisects $\angle VSW$	③ Def. of bisect

Elaborate

10. A student claims that any two congruent triangles must have the same perimeter. Do you agree? Explain.

yes all sides are the same so the perimeters will be the same

11. If $\triangle PQR$ is a right triangle and $\triangle PQR \cong \triangle XYZ$, does $\triangle XYZ$ have to be a right triangle? Why or why not?

yes b/c all the angles must be $\cong$ corresponding

12. **Essential Question Check-In** Suppose you know that pentagon $ABCDE$ is congruent to pentagon $FGHJK$. How many additional congruence statements can you write using corresponding parts of the pentagons? Explain.

10 $\cong$ statements, 5 sides + 5 angles

Homework

pg 127; 1-14,16

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