

6.5

Algebra 2

Characteristics of Rational Functions

Parent function: $f(x) = \frac{1}{x}$

D: $\{x | x \neq 0\}$

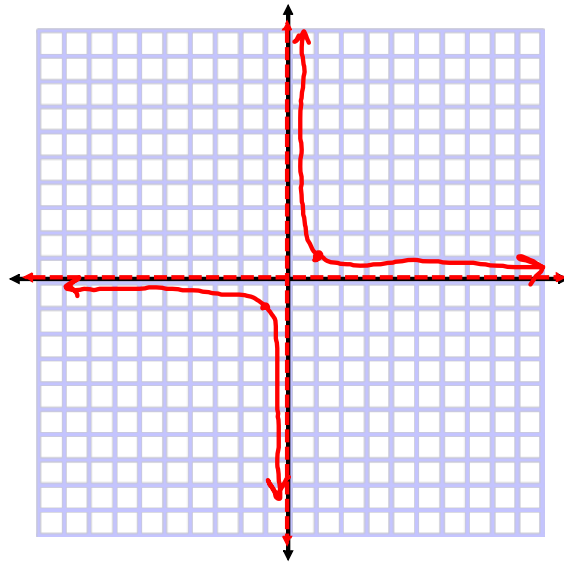
R: $\{y | y \neq 0\}$

x-intercept:

y-intercept:

VA:

HA:



Rational function: $f(x) = \frac{1}{x-h} + k$

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Finding Vertical and Horizontal Asymptotes

Vertical Asymptotes:

Set the denominator equal to zero and solve for x .

Horizontal Asymptotes:

$y=k$ for $f(x) = \frac{1}{x-h} + k$

ex) Find the Asymptotes for the given function.

$$f(x) = \frac{1}{x-3} + 7$$

VA: $x=3$ HA: $y=7$

Determine all the asymptotes, domain, range, and intercepts that the function has

1) $f(x) = \frac{1}{x-4} + 1$

VA: $x=4$ D: $\{x|x \neq 4\}$
 HA: $y=1$ R: $\{y|y \neq 1\}$
 x-int: $(3, 0)$ y-int: $(0, \frac{3}{4})$

$0 = \frac{1}{x-4} + 1$
 $-1 = \frac{1}{x-4}$
 $(x-4) \cdot -1 = \frac{1}{x-4} \cdot x-4$
 $-x+4 = 1$
 $-x = -3$
 $x = 3$
 $y = \frac{1}{0-4} + 1 = -\frac{1}{4} + 1 = \frac{3}{4}$

2) $f(x) = \frac{1}{x} - 6$

VA: $x=0$ D: $\{x|x \neq 0\}$
 HA: $y=-6$ R: $\{y|y \neq -6\}$
 x-int: $(\frac{1}{6}, 0)$ y-int: None

$0 = \frac{1}{x} - 6$
 $+6 = \frac{1}{x}$
 $x \cdot 6 = \frac{1}{x} \cdot x$
 $\frac{6x}{6} = \frac{1}{6}$
 $x = \frac{1}{6}$
 $y = \frac{1}{0} - 6$
 undefined
 so no y-int.

3) $f(x) = \frac{1}{x+1}$

VA: $x=-1$ D: $\{x|x \neq -1\}$
 HA: $y=0$ R: $\{y|y \neq 0\}$
 x-int: none y-int: $(0, 1)$

$0 = \frac{1}{x+1}$
 $0 = 1$
 false
 so no x-int
 $y = \frac{1}{0+1} = \frac{1}{1} = 1$

Try this:

Determine all the asymptotes, domain, range, and intercepts that the function has.

$f(x) = \frac{1}{x-7} + 2$

VA: $x=7$ HA: $y=2$
 D: $\{x|x \neq 7\}$ R: $\{y|y \neq 2\}$
 X-int: $(\frac{13}{2}, 0)$ y-int: $(0, \frac{13}{7})$

$0 = \frac{1}{x-7} + 2$
 $-2 = \frac{1}{x-7}$
 $-2(x-7) = 1$
 $-2x + 14 = 1$
 $-2x = -13$
 $x = \frac{13}{2}$

$y = \frac{1}{0-7} + 2 = -\frac{1}{7} + 2 = \frac{16}{7} = \frac{13}{7}$