Math Analysis
P.7 Equations
(linear, quadratic, rational, square root, absolute value, radical)

$$\frac{28}{1}\left(\frac{x-3}{4}=\frac{5}{14}-\frac{x+5}{7}.\right)$$

$\Rightarrow \frac{28(x-3)}{4} = \frac{5\cdot 28}{14} - \frac{28(x+5)}{7}$

$7(x-3) = 10 - 4(x+5)$

$7x-21 = 10 - 4x - 20$

$+4x \quad +21 \quad +21 \quad +4x$

$\frac{11x}{11} = \frac{11}{11}$

$\boxed{x=1}$

Solving a Rational Equation

$$\left(\frac{6}{x+3}-\frac{5}{x-2}=\frac{-20}{x^2+x-6}.\right)\cdot\frac{(x-2)(x+3)}{1}$$

$(x-2)(x+3)$

$\frac{6(x-2)\cancel{(x+3)}}{\cancel{x+3}} - \frac{5\cancel{(x-2)}(x+3)}{\cancel{x-2}} = \frac{-20\cancel{(x-2)}\cancel{(x+3)}}{\cancel{(x-2)}\cancel{(x+3)}}$

$6(x-2) - 5(x+3) = -20$

$x \neq 2, -3$

$6x-12 - 5x - 15 = -20$

$x - 27 = -20$

$x = 7$

Solving a Formula for a Variable

Solve for q: $\left(\frac{1}{p}+\frac{1}{q}=\frac{1}{f}\right)\cdot\frac{pqf}{1}$ = $\frac{\cancel{p}qf}{\cancel{p}}+\frac{p\cancel{q}f}{\cancel{q}}=\frac{pq\cancel{f}}{\cancel{f}}$

$$qf+pf=pq$$
$$-qf \qquad\qquad -qf$$
$$pf=pq-qf$$
$$\frac{pf}{p-f}=\frac{q\cdot\cancel{(p-f)}}{\cancel{p-f}}$$
$$\boxed{q=\frac{pf}{p-f}}$$

You Try:

$$\left(\frac{-1}{x+1}=\frac{1}{\underset{3(x+1)}{3x+3}}-\frac{2}{x-4}\right)\frac{3(x+1)(x-4)}{1}$$

$$\frac{-1(3)\cancel{(x+1)}(x-4)}{\cancel{x+1}}=\frac{\cancel{3(x+1)}(x-4)}{\cancel{3(x+1)}}-\frac{2(3)(x+1)\cancel{(x-4)}}{\cancel{x-4}}$$

$$-3(x-4)=x-4-6(x+1)$$
$$-3x+12=x-4-6x-6$$
$$-3x+12=-5x-10$$
$$2x=-22$$
$$x=-11$$

Solve the formula for the specified variable.

$$\frac{S}{\pi L}=\frac{\pi(r_1+r_2)L}{\pi L}\text{, for } r_2$$

$$\frac{S}{\pi L}=r_1+r_2$$
$$-r_1 \qquad -r_1$$

$$\boxed{\frac{S}{\pi L}-r_1=r_2}$$

The **absolute value** of *x* describes the distance of *x* from zero on a number line. To solve an absolute value equation, we rewrite the absolute value equation without absolute value bars.

If *c* is a positive real number and *u* represents an algebraic expression, then is equivalent to *u* = c or *u* = – c.

$|u| = c$

$u = c$ & $u = -c$

$$4|1-2x|-20=0.$$

$+20 \quad +20$

$\frac{\cancel{4}|1-2x|}{\cancel{4}} = \frac{20}{4}$

$|1-2x| = 5$

$1-2x=5$

$-1 \quad -1$

$\frac{-2x}{-2} = \frac{4}{-2}$

$x=-2$

$1-2x=-5$

$-1 \quad -1$

$\frac{-2x}{-2} = \frac{-6}{-2}$

$x=3$

$4|1-2(-2)|-20=0$

$4|1+4|-20$

$4|5|-20$

$4 \cdot 5-20$

$20-20$

0 ✓

$4|1-2\cdot 3|-20=0$

$4|-5|-20=0$

$4 \cdot 5-20$

$20-20=0$ ✓

A **quadratic equation** in *x* is an equation that can be written in the **general form**

$$ax^2+bx+c=0,$$

where *a*, *b*, and *c* are real numbers, with $a \neq 0$.

A quadratic equation in *x* is also called a **second-degree polynomial equation** in *x*.

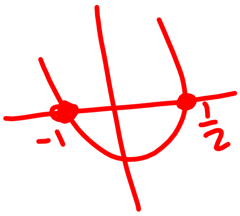
To solve a quadratic equation by factoring, we apply the **zero-product principle** which states that:

If the product of two algebraic expressions is zero, then at least one of the factors is equal to zero.

If $AB = 0$, then $A = 0$ or $B = 0$.

Solving a Quadratic Equation by Factoring

1. If necessary, rewrite the equation in the general form $ax^2 + bx + c = 0$ moving all nonzero terms to one side, thereby obtaining zero on the other side.
2. Factor completely.
3. Apply the zero-product principle, setting each factor containing a variable equal to zero.
4. Solve the equations in step 3.
5. Check the solutions in the original equation.



$$2x^2 + x = 1.$$

$-1 \quad -1$

$2x^2 + x - 1 = 0$

$(2x - 1)(x + 1) = 0$

$2x - 1 = 0 \qquad x + 1 = 0$

$x = \frac{1}{2} \qquad x = -1$

$2(\frac{1}{2})^2 + \frac{1}{2} = 1$
$2(\frac{1}{4})$
$\frac{1}{2} + \frac{1}{2} = 1$ ✓

$2(-1)^2 - 1 = 1$
$2(1) - 1$
$2 + -1 = 1$ ✓

Example: Solving Quadratic Equations by the Square Root Property

$$3x^2 - 21 = 0$$

$+21 \qquad +21$

$\frac{3x^2}{3} = \frac{21}{3}$

$\sqrt{x^2} = \sqrt{7}$

$\boxed{x = \pm\sqrt{7}}$

$3(\sqrt{7})^2 - 21 = 0$
$3 \cdot 7$
$21 - 21 = 0$ ✓

Completing the Square

If $x^2 + bx$ is a binomial, then by adding $\left(\frac{b}{2}\right)^2$, which is the square of half the coefficient of x, a perfect square trinomial will result. That is,

$c = \left(\frac{b}{2}\right)^2$

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2.$$

Example: Solving a Quadratic Equation by Completing the Square

$$x^2 + 4x - 1 = 0.$$

$x^2 + 4x + 4$

$(x+2)(x+2)$

$c = \left(\frac{4}{2}\right)^2 = 4$

$+1 \quad +1$

$x^2 + 4x + \underline{4} = 1 + \underline{4}$

$\sqrt{(x+2)^2} = \sqrt{5}$

$x + 2 = \pm\sqrt{5}$

$-2 \quad -2$

$\boxed{x = -2 \pm \sqrt{5}}$

$-2 + \sqrt{5}$

$-2 - \sqrt{5}$

Example: Solving a Quadratic Equation Using the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\overset{a}{2}x^2 + \overset{b}{2}x \overset{c}{-1} = 0.$$

$$x = \frac{-2 \pm \sqrt{\overset{4}{2^2} - [4(2)(-1)]}}{2(2)}$$

$$\frac{-2 \pm \sqrt{4+8}}{4} = \frac{-2 \pm \sqrt{12}}{4} \quad 4\cdot 3$$

÷ out GCF

$$= \frac{-2 \pm 2\sqrt{3}}{4} = \boxed{\frac{-1 \pm \sqrt{3}}{2}}$$

The Discriminant

We can find the solution for a quadratic equation of the form $ax^2 + bx + c = 0$ using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The **discriminant** is the quantity $b^2 - 4ac$ which appears under the radical sign in the quadratic formula. The **discriminant** of the quadratic equation determines the number and type of solutions.

If the discriminant is positive, there will be two unequal real solutions. $\sqrt{23}$

If the discriminant is zero, there is one real (repeated) solution. $\sqrt{0}$

If the discriminant is negative, there are two imaginary solutions. $\sqrt{-5}$

Example: Using the Discriminant

Compute the discriminant of $\overset{a}{3}x^2 - \overset{b}{2}x + \overset{c}{5} = 0$.

What does the discriminant indicate about the number and type of solutions?

$b^2 - 4ac$

$(-2)^2 - 4(3)(5)$

$4 - 60$

-56

2 imaginary sol'n's

You Try:

. Solve using the quadratic formula.

$2x^2 = 6x - 1$

$2x^2 - 6x + 1 = 0$

$\frac{6 \pm \sqrt{36 - 4(2)(1)}}{2 \cdot 2}$

$\frac{6 \pm \sqrt{28}}{4}$ $\quad 4 \cdot 7$

$\frac{6 \pm 2\sqrt{7}}{4}$

$\frac{3 \pm \sqrt{7}}{2}$

Solve by factoring.

$3(2x^2 + 3x) = 4(2x + 3)$

$6x^2 + 9x = 8x + 12$

$6x^2 + x - 12 = 0$

$(3x - 4)(2x + 3) = 0$

$3x - 4 = 0 \quad 2x + 3 = 0$

$\boxed{4/3 \ \& \ -3/2}$

$|2x - 4| + 3 = 8$

$\quad -3 \quad -3$

$|2x - 4| = 5$

$2x - 4 = 5 \ \& \ 2x - 4 = -5$

$2x = 9 \qquad 2x = -1$

$\boxed{x = \frac{9}{2} \ \& \ x = -\frac{1}{2}}$

Radical Equations

1. If necessary, isolate one radical on one side of the equation.
2. Raise both sides of the equation to the *n*th power.
3. Solve. If this equation still contains radicals, repeat steps 1 and 2.
4. Check all solutions in the original equation.

$$\sqrt{x+3}+3=x.$$

$$-3 \qquad -3$$

$$\left(\sqrt{x+3}\right)^2=(x-3)(x-3)$$

$$x+3=x^2-6x+9$$

$$-x \quad -3 \qquad -x \quad -3$$

$$0=x^2-7x+6$$

$$0=(x-6)(x-1)$$

$$x-6=0 \qquad x-1=0$$

$$x=6, \quad \cancel{x=1}$$

You Try:

$$\left(\sqrt{6x-2}\right)^2=(x+1)^2 \; (x+1)$$

$$6x-2=x^2+2x+1$$

$$-6x+2 \qquad -6x+2$$

$$0=x^2-4x+3$$

$$(x-3)(x-1)$$

$$x=3, \quad x=1$$

$$\sqrt{6\cdot 3-2}=3+1$$

$$\sqrt{16}=4 \checkmark$$

$$\sqrt{6\cdot 1-2}=1+1$$

$$\sqrt{4}=2 \checkmark$$

Homework

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