Math Analysis

P.9 Linear Inequalities & Absolute Value Inequalities

## Interval Notation:

The **open interval** $(a,b)$ represents the set of real numbers between, but not including, $a$ and $b$.

$$(a,b)=\{x|a<x<b\}$$

The **closed interval** $[a,b]$ represents the set of real numbers between, and including, $a$ and $b$.

$$[a,b]=\{x|a\le x\le b\}$$

The **infinite interval** $(a,\infty)$ represents the set of real numbers that are greater than $a$.

$$(a,\infty)=\{x|x>a\}$$

The **infinite interval** $(-\infty,b]$ represents the set of real numbers that are less than or equal to $b$.

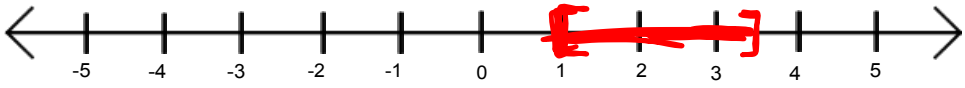
$$(-\infty,b]=\{x|x\le b\}$$

Summary: Parentheses indicate endpoints that are not included in an interval. Square brackets indicate endpoints that are included in an interval. Parentheses are always used with ∞ or -∞.

## Example: Using Interval Notation

Express the interval in set-builder notation and graph:

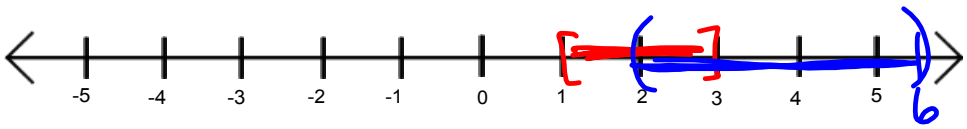
$[1, 3.5]$ $\{x \mid 1 \le x \le 3.5\}$




$(-\infty,-1)$ $\{x \mid x < -1\}$ or $\{x \mid -\infty < x < -1\}$

## Example: Finding Intersections and Unions of Intervals

Use graphs to find the set: $[1,3] \cap (2,6)$.




$(2,3]$

## Solving Linear Inequalities in One Variable

One rule to remember: **when we multiply or divide both sides of an inequality by a negative number, the direction of the inequality symbol is reversed.**

Solve and graph the solution set on a number line:
and write answer in interval notation.

$2 - 3x \leq 5$

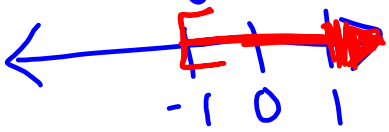
$-2 \quad -2$

flip it!

$\frac{-3x}{-3} \leq \frac{3}{-3}$

$x \geq -1$

$[-1, \infty)$



# Example: Solving a Compound Inequality

Solve and graph the solution set on a number line:
and write answer in interval notation.

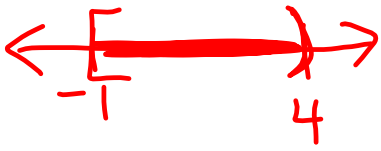
$$1 \le 2x+3 < 11.$$

$-3 \quad -3 \quad -3$

$$\frac{-2}{2} \le \frac{2x}{2} < \frac{8}{2}$$

$$-1 \le x < 4$$

$[-1, 4)$




## Solving an Absolute Value Inequality

If $u$ is an algebraic expression and $c$ is a positive number,

1. The solutions of $|u| < c$ are the numbers that satisfy $-c < u < c$. (u > -c and u < c)
2. The solutions of $|u| > c$ are the numbers that satisfy $u < -c$ or $u > c$.

Solve and graph the solution set on a number line:
and write answer in interval notation.

$$18 < |6-3x|.$$

$|6-3x| > 18$

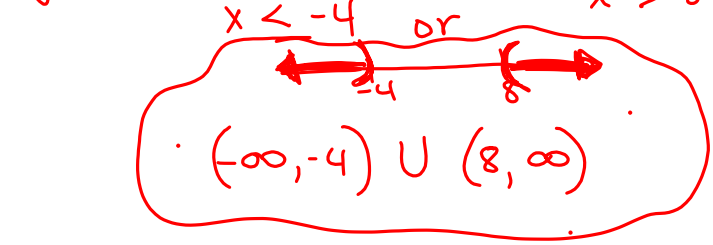
$6-3x > 18$ or $6-3x < -18$

$-6 \quad -6 \qquad -6 \quad -6$

flip it

$\frac{-3x}{-3} > \frac{12}{-3}$ $\qquad$ $\frac{-3x}{-3} < \frac{-24}{-3}$

$x < -4$ or $x > 8$




$(-\infty, -4) \cup (8, \infty)$

flip it → $\frac{-2|4x-5|}{-2} > \frac{-16}{-2}$

$|4x-5| < 8$ compound &

$-8 < 4x-5 < 8$

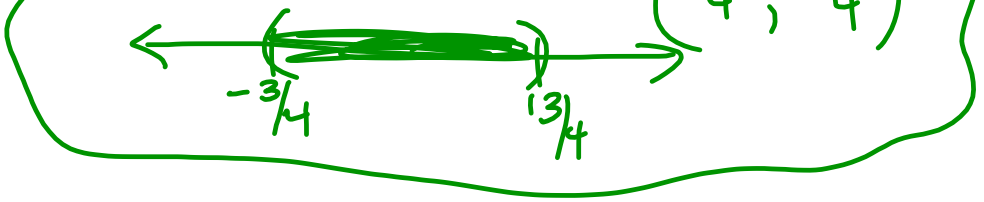
$+5 \quad +5 \quad +5$

$-3 < 4x < 13$

$\frac{-3}{4} < \frac{4x}{4} < \frac{13}{4}$

$-\frac{3}{4} < x < \frac{13}{4} = 3\frac{1}{4}$

$\left(-\frac{3}{4}, \frac{13}{4}\right)$




# Homework

pg. 131; 4, 10, 16, 22, 30-46e, 50-56e, 62-70e, 78-82e, 90, 92

ec: 48, 98, 110, 112